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THE MATHEMATICS TEACHER

Volume XLIV Number 6

A Mathematics Program for the Able*

By G. Baley Price
University of Kansas, Lawrence, Kansas

1. *Introduction.* In 1946 Herbert Hoover [10, pp. 436–437] said, "It is dinned into us that this is the century of the common man. . . . But if we are to have leadership in government, in science, in education, in the professions and in the home, we must find and train some *uncommon* men and women." The able and the gifted—once the primary concern of our schools, colleges, and universities—are now largely a forgotten group as a result of the growth of mass education. There are encouraging signs of improvement, however. Last year the Educational Policies Commission issued a report [6] entitled *Education of the Gifted* which considers the gifted student and his importance to society, and which recommends ways and means of identifying and educating the gifted. The present paper may be considered an effort to examine and interpret, in the special field of mathematics, the general conclusions and recommendations of this report. The paper begins with a background of facts; recommendations follow.

2. *A need for mathematicians.* The nation is desperately in need of mathematicians for its defense. Dr. Curtiss of the National Bureau of Standards recently said [3]: "You will recall that it has been said that World War I was a chemists' war and that World War II was a physicists' war. There are those who say that the next world war, if one should occur, will be a mathematicians' war." The instruments of modern warfare—the atomic bomb, radar, guided missiles, aeroplanes, and so on—are so complex and expensive that mathematical analysis must be used to the utmost in their design, development, and production. Mathematicians are required also for waging war even after the equipment has been built. *Operational research* or *operations analysis*, which has been defined [1, p. 4] by Dr. Kittell as "a scientific method for providing executives with a quantitative basis for decisions," and which came into being during World War II, employed many mathematicians and is seeking more at the present time. In order to secure sufficient mathematical talent to carry on World War II, most of the universities were stripped even of their pure mathematicians, and fundamental research and the training of young mathematicians were stopped.

But mathematics is needed also in time of peace. Dr. Fry [7] in 1940 traced the applications of mathematics in the communications industry, in the petroleum industry, and in several others; and the demand for mathematicians in industry

* A paper read at the 29th Annual Meeting of The National Council of Teachers of Mathematics in Pittsburgh, Pennsylvania on March 31, 1951.

Numbers in square brackets refer to the bibliography at the end.

is much greater now than it was then. The Industrial Mathematics Society was founded in May, 1949, and had attained a membership, largely in the Detroit area, of 200 by October, 1950. There are important applications of statistics to quality control, sampling procedures, and to other problems in the industrial field. Statisticians are needed for important work in the social and biological sciences, including economics, psychology, medicine, agriculture, and so on. Important mathematical work is being done in the field of biology already [16, 24], and there are need and prospect for much more. Rafferty, a biometrician, recently said, [15, p. 549] "Biologists should become more theoretical; mathematicians should become more biological."

3. *The present situation: a shortage of mathematicians.* Although there is a great need for mathematicians and statisticians, there is also a great shortage of both. An article in *Fortune Magazine* stated [18, p. 107]:

"There is at present an acute shortage of scientists. The shortage is sharper in some branches (physics, mathematics) than in others (geology), greater in some areas (basic research) than in others (applied or industrial research). But the shortage is widespread and unique in U. S. history, a product of wartime draft policies that for the duration practically halted the creation of young scientists, and a postwar expansion in scientific research beyond all expected bounds."

Although the United States occupies a leading position in science at the present time, it is well to remember that Europe held the lead up to World War II. The truth of this statement is well indicated by the record of the awards of Nobel Prizes. A recent summary [8] shows that, of the 124 prizes (to 158 persons) awarded in physics, chemistry, and physiology (there is no Nobel Prize in mathematics) up to the end of 1949, $20\frac{1}{2}$ (29 persons) went to the United States while $33\frac{1}{2}$ (38 persons) went to Germany, 20 (28 persons) went to Great Britain, $11\frac{1}{2}$ (16 persons) went to France, and the remainder were scattered among numerous smaller countries. It is well known that the United States was almost wholly dependent on England, France, and Germany for fundamental research in applied mathematics up to the beginning of World War II.

Another aspect of our present position should be noted carefully. In spite of our present shortage of mathematicians and statisticians, it is generally agreed that the United States ranks first in mathematics in the world. But it must be remembered that the United States reached its present position of leadership only with the help of a large number of mathematicians who were imported from abroad. Professor Dresden [4] has listed the names of 130 mathematicians and mathematical physicists who came to the United States during the years 1933–1942. Many others came before 1933 or after World War II. There is no stockpile of mathematicians, and access to our foreign sources of supply might be cut off completely at any time.

4. *Highest ability required.* Perhaps you are puzzled. Why should there be a shortage of mathematicians when our colleges and universities are crowded, and when instruction in mathematics is given to hundreds and thousands? The answer is simple: in mathematics, as in other fields of science, it is only the exceptional and highly gifted individual who can be a creative scholar.

Conant said [2; 5, p. 413]: "Advance in science depends upon the number of first-class men engaged in scientific work, and no possible number of second-rate men will provide a substitute. . . . We must see to it that, as far as humanly possible, all the potential talent of the country . . . is recognized at an early age and given adequate opportunity."

An article in *Fortune Magazine* states [18, p. 108]: "The significance of these figures lies in the fact that there are all together only some 140,000 people engaged in scientific research, development,

and teaching in the U. S., and of these only about 25,000 hold doctors' degrees in the physical, biological, and agricultural sciences. The whole scientific and technological progress of the country, therefore rests upon a small fraction of 1 per cent of the population. And the doctorate group, representing the vanguard of science, could easily be assembled in one large hall. Strictly speaking, it is mainly the doctors who are the scientists, men who advance the frontiers of knowledge into the unknown."

Professor André Weil, a leading pure mathematician, has written [22, p. 304]: "Moreover, mathematical talent usually shows itself at an early age; and the workers of the second rank play a smaller role in it than elsewhere, the role of a sounding board for sounds in whose production they had no part."

Dr. T. C. Fry, writing on industrial mathematics [7, p. 270] said: "In other words, the mathematician in industry, to the extent to which he functions as a mathematician, is a consultant, not a project man. . . . He must be a man of outstanding ability. No one wants the advice of mediocrity. Among industrial mathematicians there is no place for the average man."

Finally, the universities receive many calls for mathematicians of the highest ability and extensive training (to the Ph.D. degree and beyond), but relatively few calls for holders of A.B. degrees with majors in mathematics.

5. *The young create mathematics.* It is well known that mathematics is created largely by the young. Weil's statement concerning the early appearance of mathematical talent has been quoted already. As examples, many will recall Abel and Galois, two of the really great mathematicians of the early part of the nineteenth century. Abel died at age 26 and Galois at age 20.

G. H. Hardy, one of the leading pure mathematicians of the first half of the twentieth century, has had much to say on mathematics and youth [9, pp. 10–13]. He asserted that, although he still had many good ideas at age sixty, he no longer had the strength necessary to develop them and carry them through to completion.

An article in *Fortune Magazine* states [18, p. 111]: "If the most creative years of a large number of scientists are charted on a graph, the peak comes about age thirty-two. But this, too, varies with the sciences. Mathematicians generally bud early, often before twenty-five; physicists come later, between thirty and forty; and biologists, whose science depends on a mass of patient observation, usually are most productive late in life."

Finally, it must be pointed out that, in a large majority of cases at the present time, the training of a mathematician in the United States begins in his senior year in college or in his first year as a graduate student, that is, at age twenty-one or twenty-two. In most cases the student takes no course before this time which is an adequate test of either his interest or his ability in mathematics. A reading knowledge of French and German is required for successful work in mathematics, and a reading knowledge of Russian is now highly desirable. In a large majority of cases, the beginning graduate student in mathematics in the United States is unable to read any foreign language.

6. *What is the mathematicians' apology?* In the effort to teach mathematics to the gifted, and especially to develop creative mathematicians, it will be helpful to understand the reasons which lead people to undertake a serious study of mathematics and to devote their lives to mathematical research. Some information is available, but it is surprisingly difficult to give an answer that satisfies even the mathematicians.

There are at least a few who become mathematicians because, they say, they find that mathematics offers an easier means of earning a living than any other field they know. But the meager salary of

the mathematician [18, pp. 109–110] cannot be considered an inducement to devote one's life to mathematical research.

Professor Henry D. Smyth, author of the report on the atomic bomb and member of the Atomic Energy Commission, has said [17, p. 427]: "The primary motive that leads men to study the behavior of nature is intellectual curiosity, a very personal desire to understand the 'how' of things."

In discussing the question "What makes a scientist?", the article in *Fortune Magazine* [18, p. 111] says: "One explanation is that science is one of the true vocations—a calling—and the best scientists become scientists because they cannot help themselves. By far the leading motive is the freedom offered by the sciences for the constructive exercise of the mind upon the material world, a motive that received first mention by 53 per cent of scientists surveyed for the Steelman report. Science is probably the only profession that not only encourages radical thought, in the sense of an experimental search for new principles below the surface of the accepted, but reserves its highest honors for radical ideas. Nearly 50 per cent of all scientists surveyed, regardless of where employed, picked university work as ideal because of its greater freedom."

But to these statements it is necessary to add another. G. H. Hardy, himself a mathematician of outstanding accomplishments, wrote a small book [9] to explain why he devoted his life to mathematical research. His explanation has not satisfied many people. Recently Sir Edmund Whittaker, also an important mathematician, has written [23, p. 42] as follows:

> A small book published in 1940 by Hardy under the title *A Mathematician's Apology* attracted a great deal of attention. "Why," he asked, "is it really worth while to make a serious study of mathematics? What is the proper justification of a mathematician's life?" Hardy's own answer was that "if a man has any genuine talent, he should be ready to make almost any sacrifice in order to cultivate it to the full," and he justified his choice of a career by his consciousness of possessing great mathematical ability. But the critics asked: "If Hardy had been conscious of great agility in climbing drainpipes, ought he to have become a cat-burglar? If he had found that he had a great attraction for the other sex, ought he to have set up as a Don Juan?" The fact is that the problem he proposed really belongs to moral philosophy, and Hardy, who was a stranger to moral philosophy, had no principles available to deal with it. Curiously enough, I do not remember ever having seen a sustained argument by any author which, starting from philosophical or theological premises likely to meet with general acceptance, reached the conclusion that a praiseworthy ordering of one's life is to devote it to research in mathematics. Whatever the philosophical situation may be, it must be admitted that the 20th century would not have been the same without its mathematicians.

7. *A mathematics program for the gifted.* The above facts will now serve as a background for certain specific recommendations concerning the education of the gifted in mathematics. The "highly gifted" are understood to be [6, p. 43] the top one per cent with respect to intellectual capacity (that is, roughly, individuals with an I.Q. above 137); similarly, the "moderately gifted" are understood to be the individuals who fall within the top 10 per cent but below the top one per cent (that is, individuals with an I.Q. between 120 and 137). The recommendations may be grouped under five headings as follows:

(a) Identify the gifted student and, in particular, those with special ability in mathematics.

The student who is highly gifted in mathematics is usually "highly gifted" in the sense defined above. Thus the first instrument to be used in searching for the mathematically gifted is the intelligence test. The second instrument to be used is a competitive mathematics examination which has been designed to locate mathematical ability. The first of these instruments is well known, but the second probably is not.

There is convincing evidence by this time that students of high intellectual capacity—students who will far outstrip their fellows in their later intellectual and professional accomplishments—can be suc-

cessfully identified early [20]. There is evidence [5] that the efforts to select those superior in scientific aptitude have been successful. Finally there is evidence that the creative mathematician can be identified with considerable success at least by the time he graduates from high school. Professor Tibor Radó, reporting [14] on the mathematical competitions held each year for determining the winner of the Eötvös-prize founded by the Hungarian Mathematical Society in 1894, has written as follows:

> Is it possible to test by problems creative mathematical ability? Certainly it would be possible to prove the impossibility of such an attempt on the basis of psychological theories. First of all, a boy can be really able, and still be unable to solve anything in four hours, in strange surroundings. There is the great danger of discouraging this type. There are many other objections; it might be interesting, but certainly futile, to discuss them, simply because the experience of almost forty years shows conclusively that this mathematical competition *could* be made a success.

It is in order to report that three competitive mathematical examinations are held each year in the United States. The oldest of these is [21] the William Lowell Putnam Mathematical Competition, for college students, conducted by the Mathematical Association of America, and held each year since 1938 except for an interruption of three years during World War II. The next is [19] the Stanford University Competitive Examination in Mathematics, for high school seniors, which has been held each year since 1946. The newest addition to the list is the Mathematics Contest of the Metropolitan Section (of the Mathematical Association of America), for high school students, which was held for the first time in 1950. These competitive mathematical examinations are serving a useful purpose, and more high school and college students should be given an opportunity to compete each year.

(b) Educate the mathematically gifted to the limit of their abilities—in spite of the draft or war—for the sake of the safety and welfare of the nation.

Mathematical ability is a scarce item, and an adequate supply is essential for the defense of the nation. During World War II the training of scientists was essentially stopped because all students were drafted, and the nation is suffering the consequences now. The countries of Europe, where military conscription has been practiced for many years, have always been more foresighted. In Russia, in China, in Finland, in England, and in other countries of Europe the gifted student was educated even during World War II—not drafted.

Our policies concerning the drafting of students and universal military service are again the subject of much debate. There has been support by President Conant of Harvard, the American Association of Universities, the Department of Defense, General Eisenhower, and the Senate for a plan which would include college training for perhaps 75,000 selected students annually. Because of the present scarcity of mathematical talent and the long time required for its development, because mathematics is an activity of youth, and because our only hope of overcoming the superior numbers of our enemies is by means of superior skill and ability, a provision to continue the training of mathematicians and other scientists seems absolutely necessary.

Dr. Henry D. Smyth, speaking to the American Association for the Advancement of Science at its meeting in Cleveland on December 28, 1950, recommended the formation of a Scientific Service Corps for scientists of military age and of a Student Science Corps for the scientifically promising student. The Student Science Corps would enroll science students of high intelligence and promise in their freshman year. They would stay in through graduate school training if they were intelligent, imaginative, original, and good performers. Others who were only technically competent would be released to the military and civilian life after finishing college, and students would be "continually

weeded out." This Corps would be under the general jurisdiction of the National Science Foundation.

Dr. Smyth's recommendation recognizes the obligation of all to serve their country in time of need, but adds the important principle that the gifted, like all others, should serve in the position in which they can make their greatest contribution.

The National Science Foundation is now in existence. Through scholarships and other aids, it will undoubtedly lend valuable assistance to carrying out the recommendation to educate the mathematically gifted to the limit of their abilities.

(c) Counsel all students, including the mathematically gifted, to study the mathematics they will need in their later work.

This recommendation may appear to be trivial, but there is reason to believe that it is one of the most important of the five. Many students, upon entering college, are surprised to learn that their high school preparation in mathematics is inadequate. Some report that their schools did not give the necessary courses. Others say, "But nobody told me that I would need these courses."

No better specific recommendations (even for the gifted in mathematics) can be given than those in *Education of the Gifted* [6, pp. 63–64]:

For these reasons we recommend that there be included in the secondary-school program of nearly every *highly* gifted student the following:

(1) A *foreign language;* studied for a long enough time and with sufficient intensity to achieve, at least, reading mastery.

(2) Advanced *mathematics;* certainly through advanced algebra, probably through trigonometry, possibly through calculus.

(3) Additional study of the *social studies,* with emphasis on history, beyond the amount taken by the typical high-school student.

It has been said that "nearly every highly gifted student" should follow the program outlined above. There are two exceptions. (1) The youth who is highly gifted in music or art as well as in general intelligence may safely omit advanced mathematics but not social studies or foreign language. (2) The highly gifted youth who suffers from physical disability or from social or emotional maladjustment in adolescence to the extent that intensive academic work for him in high school may be either impossible or undesirable should have his high-school program modified accordingly.

With these exceptions, we recommend the above program for *all* highly gifted youth. We say this with a full realization that some of them may have relatively little use for some parts of the program in their later study and life work. Thus, the young man or woman who studies law or public administration may make little use of his mathematical training and the research chemist may have little use for a broad background in history (as far as his professional work is concerned). But, because all avenues for later specialization should be kept open for the gifted youth until he is in his upper years of college, he should not be limited in his choice by lack of previous preparation. It is for this reason that we recommend the full program for all highly gifted youth. The discredited claim that study of foreign language and mathematics are valuable because they "discipline the mind" has no bearing on the matter.

There are two further and related considerations. In the first place, the highly gifted student who has completed a substantial amount of advanced academic work in high school, along the lines here recommended, has a better basis for choosing his area of specialization in college and he can make a better decision on this matter at an earlier stage in his college work. In the second place, he is spared the necessity of taking valuable time from his more mature student years to equip himself with the tools and basic information that are essential to advanced study and to creative achievement in scholarship and the intellectual professions.

Many *moderately* gifted students could also profit from more social studies, advanced mathematics, and foreign language in high school; but in their case the need is neither so clear nor so compelling as it is in the case of the highly gifted.

(d) Stimulate the interest of the mathematically gifted by means of good teaching, competitive mathematical examinations, and recognition of high accomplishment.

An important function of the teacher of the mathematically gifted is the stimulation of interest. Even the most gifted student will be repelled if he encounters only trivial courses and dull teachers. The teacher must have, above all else, a knowledge of, and a genuine interest in, mathematics.

The stimulation of interest is probably more difficult in mathematics than in the other sciences. One of the reasons is that

modern mathematics, including the modern applications, is almost unknown to high school teachers of mathematics. The high school student can build radios and model airplanes, operate television, and so on, but he is fortunate if his progress in mathematics carries him as far as the calculus—a subject now almost three hundred years old! Even the college textbooks on calculus would look quite familiar to the student of a hundred years ago. How different from the situation in physics, chemistry, engineering, medicine, and so on! The same differences exist in the publicity concerning current developments in the various fields. The newspapers and magazines usually find mathematics not understandable and hence gently ridicule it. The treatment given the other sciences is very different. It is worth calling attention to one exception in the treatment of mathematics: recently the *Scientific American* has published a number of significant articles on mathematics. The lack of publicity and popular knowledge of mathematics certainly places an additional obligation on the teacher. It is recorded [18, p. 112] that "many [chemists] were originally lured into the science by two books—*The Romance of Modern Chemistry* and Stokley's *Science Remakes Our World*—which for many years were about the only scientific books available in small-town libraries."

Lacking some of the aids available in other fields, the teacher of mathematics can nevertheless employ the stimulation of competition. Large numbers of students [11, 19, 21] enter the competitive mathematical examinations that are held each year in the United States, and it seems certain that they help to increase the number of students who make a serious study of mathematics.

Finally, recognition should be given for outstanding achievement in all fields, including mathematics. Every community in the nation honors its athletes, and considerable recognition is given to students in music and art. Students in mathematics are usually ignored. Recognition may be based on performance in a competitive mathematical examination (as it usually is in athletics and music), and at least one examination [11] appears to have been designed so that it gives recognition to a large number of participants.

(e) Instruct, using the best teachers, the most approved courses, and all the care and solicitude possible in each individual case.

Special provision should be made for the mathematically gifted whenever possible. Remembering that mathematics is an activity of the young, and that some investigators [20] find *acceleration* to be quite desirable in the case of the gifted generally, careful consideration should be given to this provision in the case of a student who shows promise in mathematics. At least in the later years of high school, *grouping* on the basis of ability seems highly desirable; the instruction that is required for the gifted is not appropriate for the average student. The grouping will probably be brought about to a considerable extent by means of the *elective courses* which have been recommended for the gifted, and which will not be taken by the average student. *Enrichment* should be employed whenever possible. The full development of the gifted requires a great deal of attention to the individual.

The mathematics teacher of the mathematically gifted should know mathematics; a knowledge of teaching methods is not sufficient. This fact places an obligation on the colleges and universities to train good teachers. The university training of a teacher should awaken an appreciation of the beauty of mathematics, include courses which are not trivial and dull, and involve contact with some phases of modern mathematics. The usual teacher preparing to teach in high school takes the standard sequence through calculus and differential equations and then subjects such as modern synthetic geometry, theory of equations, and so on, because these

courses concern the subject matter of high school courses. Modern mathematics remains beyond the senior year, and the prospective teacher makes no contact with it unless he takes graduate work. It is a surprising fact that the high school and college courses in mathematics have remained essentially unchanged for a hundred years. It is possible that better and more modern courses could be devised. The University of Chicago has tried [12, 13], but it is too early to appraise the results.

Finally, the teacher of the mathematically gifted will fail if he stresses only the *usefulness* of his subject. The young are full of curiosity and easily attracted by something that is interesting and elegant. The high school student is much less attracted by an opportunity to take up the world's burdens. Hardy has said [9, pp. 59–60]: "It is not possible to justify the life of any genuine professional mathematician on the ground of the 'utility' of his work." The teacher of the mathematically gifted must not forget to present the aesthetic side of the subject.

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Misconceptions about Rationalization in Arithmetic

By J. Fred Weaver
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Within recent years the doctrine of meaning in arithmetic has received wide recognition. Much professional writing has been done in an attempt to translate sound theory into effective practice. Unfortunately, however, certain aspects of the basic doctrine have been misinterpreted. The rationalization of computational processes or skills is an important case in point.

All who subscribe to the "meaning theory" of instruction place considerable emphasis upon rationalization, and rightfully so. But all persons do not agree on its specific role as part of a broad instructional program. For example, disagreement has been observed in regard to the temporal relation between the *how* of a computational process or skill and its *why*. With apology one may ask: "*How-why*, or *Why-how*?" That is the question.

The *How-why* Point of View

There are numerous persons who advocate exclusive adherence to a *how-why* sequence: the *how* of a computational process or skill must precede the *why*. One of the most emphatic statements of this viewpoint was given a short time ago by Johnson[1] in The Mathematics Teacher. Although it may be dangerous to quote out of exact context, the writer feels that no injustice is done in citing the following statements from the Johnson article.

> The *how* of any process should be fully understood before rationalization.
>
> The *why* cannot be given before the *how* a thing is done is known because the step which we are trying to tell the reason for in a process must be known before the reason for it can be given.
>
> . . . a rationalization of a process in arithmethic is meaningless unless the *how* to do that process is understood first.
>
> Teaching the rationalization of a process before the *how* of the process is fully understood may confuse rather than clarify the process.

These statements, then, typify the basic position of persons who would answer the question, How-why or Why-how?, with an emphatic "Teach the *how*, then the *why*." This becomes an instructional policy to be followed without exception.

The present writer is not at all certain that the *how* of a process or skill must necessarily precede the *why*. In fact, a *why-how* sequence is both possible and desirable in many instances. An acceptance of the *how-why* sequence as a policy to be followed without exception is dangerous—not because of what it does, but because of what it fails to do. Unless a change is made from *how-why* to *why-how* in connection with numerous skills, an important aid to learning will be lost.

The *Why-how* Point of View

The skill of carrying in addition may be used to illustrate the feasibility and desirability of a *why-how* temporal sequence. Let us assume that this new skill first is encountered when 27 must be combined with 38.[2] *With a background of previous meaningful instruction* an attack upon this quantitative situation will, rather naturally, follow lines similar to the following:

$$\begin{array}{rcl} 38 & = & 3 \text{ tens} + \ \ 8 \text{ ones} \\ +27 & = & 2 \text{ tens} + \ \ 7 \text{ ones} \\ \hline 65 & = & 5 \text{ tens} + 15 \text{ ones, or} \\ & & 6 \text{ tens} + \ \ 5 \text{ ones, or } 65 \end{array}$$

[1] J. T. Johnson, "What Do We Mean by Meaning in Arithmetic?" The Mathematics Teacher, XLI (1948), 362–367.

[2] Some textbooks and courses of study unfortunately introduce carrying with an example having a 2-digit addend and a 1-digit addend, which truly is a higher-decade combination with bridging and does not necessitate carrying.

Undoubtedly, representative materials will be used by teacher and children to dramatize this and similar examples as previous experiences and understandings are applied to the new situation. In other examples United States money may be used to advantage. The accompanying verbalization will be in terms of "dimes" and "pennies" rather than "tens" and "ones."

At first the written algorism serves only as a symbolic record of a particular quantitative experience. Then, as a result of a variety of meaningful experiences of this nature, many children very frequently discover and formulate for themselves the more mature and efficient way to find the sum from the algorism itself—i.e., by carrying. The common crutch, indicated below, may or may not be suggested by the children:

$$\begin{array}{r} 1 \\ 38 \\ +27 \\ \hline 65 \end{array}$$

Regardless, the *how* of carrying has been discovered as a consequence of experiences associated directly with the *why* of that skill.

A comparable type of rationalizing activity is useful in connection with numerous other skills such as borrowing in subtraction or carrying in multiplication. In each instance the child learns *how* to formulate the preferred algorism through meaningful experiences which are related to the *why* of that algorism. In each instance *why* leads to a *discovery* of *how*. Thus for many skills, but not necessarily all, the *why-how* temporal sequence is definitely feasible. And not only is it feasible; it is desirable as well.

The Value of the *Why-how* Sequence

The important distinction between the products of learning and the process of learning is pertinent in this connection. Brownell[3] described this contrast and its implications very clearly some years ago. More recently, Anderson and Gates[4] have written as follows:

> By *process* is meant the way in which the learner operates in order to attain certain learning products. Process refers to the *way* in which one learns. But it also has a larger significance because process, a way of learning, itself becomes established, and a particular way of learning with its consequent meaning for a way of attacking new problems is often as important to an individual as a particular product. . . . It seems sound that the school should attempt to stimulate learning through processes which are meaningful, insightful, or problem-solving in character, because this kind of learning behavior is especially valuable for children and adults in our society.

Now consider the child who first encounters a subtraction such as $40-27$. At this initial stage for the skill in question, *process of learning* should be of primary concern. Confronted with a new situation for which he has no immediately satisfactory response, the child may make a variety of "provisional trys." He may utilize the skill of counting; he may work with the actual objects described in the basic quantitative situation; he may resort to the use of some semi-concrete or representative materials; he may work with the number symbols themselves. In any event, these types of response will differ in relevancy, maturity, and efficiency, depending largely upon the background of related experience which the child may bring to this new situation. Less relevant, mature, and efficient responses will be eliminated as they lead to more acceptable ones. If a background of meaningful experience is brought to the new situation, it is not at all uncommon to find children discovering the mechanics of the skill of borrowing in a rational way with a minimum of irrelevant and imma-

[3] William A. Brownell, "Psychological Considerations in the Learning and the Teaching of Arithmetic," *The Teaching of Arithmetic*, Chapter I. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935.

[4] G. Lester Anderson and Arthur I. Gates, "The General Nature of Learning," *Learning and Instruction*, pp. 27–28. Forty-ninth Yearbook of the National Society for the Study of Education, Part I. (Chicago: University of Chicago Press, 1950.)

ture "provisional trys." Thus it is seen that, at this initial learning stage, *why* may lead to *how*. In so doing, children gain invaluable experience in a process, or "way of learning," which will serve them well in their attack upon new quantitative situations now and later.

If the sequence is reversed, if why *follows* how (sometimes by days or months or years), it becomes increasingly difficult for initial learning experiences to be most effective in the manner described. If first we tell a child *how* to borrow in subtraction, but defer the *why* until some future time, there is grave danger that initial learning may be largely mechanistic or authoritative. The advantages to be gained from the standpoint of learning *process* in the *why-how* temporal sequence frequently may be lost when the order is reversed. This loss to the child is one for which there is no compensation. Whenever it is possible to do so, let us gain the advantage of the *why-how* sequence.

A Point of View Concerning *How-why*

Thus far, this writer has objected to the adoption of a *how-why* sequence as an instructional policy to be followed without exception. Further, an attempt has been made to show that, whenever possible, definite psychological advantage is to be gained by following a *why-how* temporal sequence. No contention has been made or implied that it is always feasible for *why* to lead to *how*. In some instructional situations it may seem virtually necessary to present the algorismic form of certain computational skills on the basis of a *how-why* sequence. In such instances, when *how-why* is selected as the course to be taken, let us be certain that the *why* is coupled with the *how* just as soon as possible or feasible. There is grave danger that *why* may follow *how* at such temporal distance that ultimate rationalization is minimized in effectiveness.

Anderson and Gates[5] make the following statement in their discussion of Transfer of Learning.

[5] *Ibid.*, p. 29.

> It seems clear that learning which is cognitive in process and control is more likely to have varied utility than learning which is mechanically associative. The most brilliant illustration of this concept is found in the development of the "meaning theory" for teaching arithmetic.

If learning is to transfer most effectively and have most varied utility, if learning is not to be "mechanically associative," it is urgent that *why* be coupled as closely as possible with *how*. In the event that the *how* of a skill does precede the *why*, let us see that rationalizations follow without unnecessary loss of time.

In a recently published research monograph Brownell and Moser[6] submit clear-cut evidence regarding the relative transfer values when the skill of borrowing in subtraction was taught meaningfully and mechanically. When subtraction examples such as $43-18$ were clearly understood and rationalized by children, there was significantly greater transfer to untaught examples such as $428-263$ than when the initial instruction had been "mechanically associative." Thus, subsequent sub-skills in borrowing were learned and taught more easily and effectively when the initial instructional experiences were rational and meaningful. Similar conditions may well exist with other skills and sub-skills of other processes. The more we defer rationalization, the less effectively may we utilize transfer of learning and the greater becomes the danger of learning and instruction which is "mechanically associative."

It has become almost axiomatic to aver that drill should *follow* understanding. So long as rationalizations are withheld, understanding will be incomplete or lacking. If we present the *how* of a skill and withhold the *why* for an unnecessary length of time, there is every possibility that drill will *precede* understanding. The dangers of premature drill are familiar to all. Let us minimize these dangers by linking *why* with *how* as soon as possible.

[6] William A. Brownell and Harold E. Moser, *Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction.* Duke University Research Studies in Education, No. 8. (Durham: Duke University Press, 1949.)

An Unfortunate Criterion

Frequently in the preceding section of this discussion the writer has stated and implied that rationalization should follow *how* "as soon as possible." There is an indefiniteness about such an emphasis that is not appealing to some persons. Rather, they would prefer a more specific criterion for determining and stating the proper time for rationalization. It is most unfortunate that in their search for such a criterion some persons have turned to the mental age concept.

Undoubtedly the reader is familiar with the work of the "Committee of Seven"[7,8] relative to the placement of arithmetic topics in terms of mental age levels. More recently, Brueckner and Grossnickle[9] have advocated a similar basis for the gradation of arithmetic processes. Finally, the following statement by Johnson[10] extends the mental age criterion to cover the matter of rationalization:

> Research would have to lead the way showing at what mental age the various arithmetic processes could be rationalized.

The reader certainly is familiar with the significant critiques of the "Committee of Seven's" work, such as those by Brownell.[11,12] These comments are equally apropos to the matter at hand: the use of mental age as the prime factor in "readiness" for rationalization. A more recent statement by Brownell and Moser[13] epitomizes the sane position to take:

> If children have the necessary ability, knowledge, and understanding, they are 'ready.' And their 'readiness' is determined, not by their age or by their grade, but by the kind of arithmetic they have had. By manipulating their arithmetic backgrounds we may shift the placement of any arithmetical topic over a range of several years and of several grades.

"Readiness" for rationalization is primarily a function of the type of quantitative experiences which children have had, and cannot be judged primarily in terms of the criterion of mental age.

The present writer feels it advantageous to call attention to further evidence, both experimental and theoretical, which shows the inadequacy and danger of the mental age criterion. First, consider Curr's[14] study of grade-placement. It is unfortunate that this scholarly piece of research has received so little publicity. In essence the study was analogous to that of the "Committee of Seven," and was concerned with the process of division by one- and two-digit divisors. The Table* on page 381 is adapted from Curr's material and shows minimum mental ages, as deduced by regression formulae, necessary to attain designated mastery standards in seven different schools.

Consider the data for divisors of 2 through 12. Notice that the minimum mental age ranged from $8-0$ in one class to $10-4$ in another class. For divisors 13 through 99, the range was from $10-9$ to $17-7$. In either case, which mental age shall be used as the minimum? The lowest? The highest? The average? In this connection Curr has stated:

> It is immediately evident . . . (from the Table) . . . that the ability to derive any specified amount of profit from instruction in an arithmetic topic is not a universal phenomenon which occurs at the same mental age in every child's development, irrespective of the condi-

[7] *Report of the Society's Committee on Arithmetic.* Twenty-ninth Yearbook of the National Society for the Study of Education, chap. XIII. 1935.

[8] *Child Development and the Curriculum.* Thirty-eighth Yearbook of the National Society for the Study of Education, Part I, chap. XVI. 1939.

[9] Leo J. Brueckner and Foster E. Grossnickle, *How to Make Arithmetic Meaningful,* chap. III. (Philadelphia: The John C. Winston Co., 1947.)

[10] *Op. cit.*, p. 366.

[11] William A. Brownell, "Readiness and the Arithmetic Curriculum." *Elementary School Journal,* Vol. 38, pp. 344–354, 1938.

[12] ———, "A Critique of the Committee of Seven's Investigations on Grade Placement of Arithmetic Topics," *Elementary School Journal,* Vol. 38, pp. 495–508, 1938.

[13] *Op. cit.*, pp. 156–157.

[14] William Curr, "Placement of Topics in Arithmetic," *Studies in Arithmetic,* Vol. II. Publications of the Scottish Council for Research in Education: XVIII, pp. 183–218. (London: University of London Press, 1941.)

* Reprinted by permission from the Scottish Council for Research in Education and the University of London Press.

Divisors	Mastery Standards	Minimum MA's by School Classes						
		#1	#2	#3	#4	#5	#6	#7
2 thru 12	83%	9–5	9–5	9–8	8–0	9–1	10–4	9–0
13 thru 99	56%	11–3	10–9	17–7	13–10	11–6	12–10	12–7

tions under which he is taught. . . . it would appear that to derive a minimum age for a topic from the results of a large number of pupils, taught, no doubt, by methods and by teachers ranging from the supremely efficient to the hopelessly inefficient, and then to impose this minimum mental age on all teachers, irrespective of their own methods and efficiency, is equivalent to . . . (having) . . . the superior teacher reduce his work to the mediocre level derived from the combined results of all methods and types of teaching, from the best to the worst.

This strong statement by Curr points clearly to the unsoundness of any attempt to set minimum or optimum points of placement for factors such as "rationalization readiness" in terms of *mental age*.

A final objection, more theoretical in nature, may be raised to the use of mental age to determine the point of rationalization for any process or skill. Within recent times men such as Wechsler have renounced completely the concept of mental age as a truly satisfactory basis for the appraisal of intellectual capacity.[15,16] The following statement from the WISC Manual[17] is most appropriate.

Mental age is considered to represent an absolute level of mental capacity, so that an MA of 6 means the same irrespective of whether it is obtained by a child of 6, 10, or 16. If all that is implied by this statement is that subjects of different ages can obtain the same test scores, it is true but so obvious. But if one also understands it to mean that their intelligence levels are identical, the statement at best is an assumption which has to be proved. In point of fact, both clinical and statistical evidence is clearly against such an assumption. Instructors in psychometrics have devoted many hours to teaching their pupils first that the MA is a *measure of mental level* (an MA of 7 is a mental level of 7 in all children attaining this rating) and then teaching them that this is not true after all (a 5-year-old with an MA of 7 has not the same kind of mind as a 10-year-old with an MA of 7).

This is but one of several reasons cited by Wechsler for the rejection of mental age as an intermediate step in the determination of IQ. This same reason throws serious doubt upon the use of mental age as a major determiner of placement for a given rationalization. For example, assume that an MA of 9—0 has been forwarded as the optimum point for rationalization of an arithmetic skill. Two children having MA's of 9—0 may have respective CA's of 8—0 and 10—0, for example. Yet, as Wechsler has stated, these two children are not similar in the sense of having the "same kind of mind." And it is highly questionable whether they are similar from the standpoint of "readiness."

The MA concept is far from being as absolute as we may unconsciously assume it to be. As a consequence, serious doubt is thrown upon the use of mental age as the primary determiner in connection with arithmetic readiness and with placement problems in general.

Recapitulation

In this discussion the writer has attempted to deal with several misconceptions concerning rationalization in arithmetic. Exception was taken to the exclusive adherence to a *how-why* temporal sequence. Advantages of the reverse sequency, *why-how*, were presented. Further, in those instances in which the *how-why* sequence may be the more feasible one to use, attention was directed to the importance of coupling *why* with *how* just as soon as possible. Finally, discussion centered around the unsoundness of using *mental age* as the primary criterion for determining readiness for rationalization.

[15] David Wechsler, *The Measurement of Adult Intelligence*, chap. 3. (Baltimore: Williams and Wilkins, 1944.)

[16] ———, *Manual: Wechsler Intelligence Scale for Children*. (New York: The Psychological Corporation, 1949.)

[17] *Ibid.*, p. 2.

The Numeral Frame—A Device for Use in the Teaching of Counting

By Donna Norton
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The numeral frame is a development from the old-fashioned bead counter, or abacus, which for a long time was considered essential for teaching beginners in arithmetic. The bead counter was a device small enough for a child to hold in his hand, while the numeral frame, as shown by Figure 1, is large enough for demonstrations before a class. In the case of the bead counter, the movable beads were permanently attached to the device, while in the numeral frame, the blocks may be removed. The beads were not numbered, but the blocks used with the numeral frame are numbered from one to one hundred. The number symbols on the blocks are large enough to be read by every child in a class of ordinary size, and it is this feature which largely determines the practical size of the numeral frame.

As shown by Figure 1, the numeral frame consists of a structure about 5 feet in height, and $2\frac{1}{2}$ feet in width, in which there are ten horizontal rails. The ten rails each have a projection, or tongue, on the upper side. Ten blocks, which have grooves to fit the tongue, are placed upon each rail. A practical size for the blocks is 2 inches long, 1 inch thick, and $1\frac{1}{4}$ inch high. The entire frame is mounted upon a base to hold it upright. The device is easily moved, and it may be placed before a class, or stood in a convenient place for use by the pupils.

The word counting, as commonly used, has two different meanings which should be carefully distinguished. When we are counting to place objects into an ordered sequence, as first, second, third, and so on, we are using number in its *ordinal* sense. When we count to determine the number of objects in a collection without reference to their order, we are using number in its *cardinal* sense.

In teaching counting, we should recognize that it is essential for the child to understand first the meaning of number in its ordinal sense, since the child must use the serial order of the number names in determining how many objects there are in a given group.

It must be recognized that mathematics is a conceptual structure in which the learning of ideas must take place in a definite order if they are to be understood. For example, the child can not understand multiplication, which is repeated addition, before he has grasped the meaning of addition.

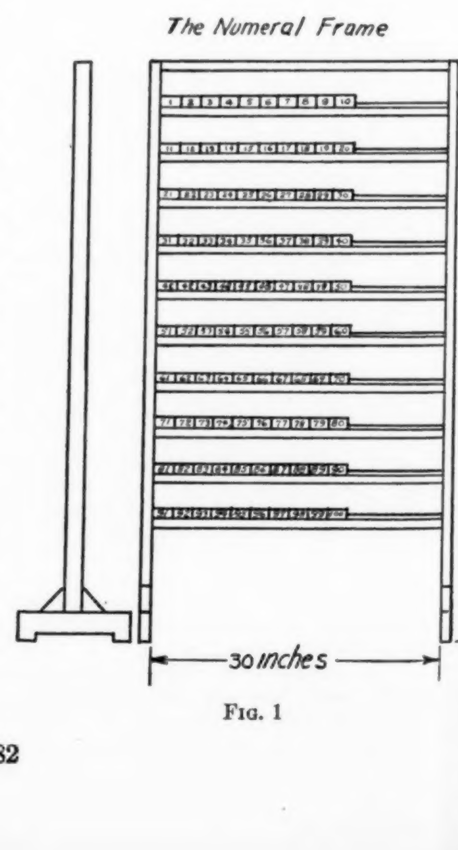

Fig. 1

Similarly, counting as a logical mathematical procedure must be mastered in a series of steps which may be designated as rote counting, enumeration, identification, reproduction, and comparison. In teaching, it is essential to preserve this order. Identification should precede reproduction because it is more easy to go from the concrete to the abstract, than from the abstract to the concrete.

The step of rote counting* is merely the learning of the number names either in a repetitive manner, or through rhymes. The object of this rote step is to give the child the number names in their correct serial order so that he may use them in making further steps. The names themselves are purely a matter of language, while the conception of serial order is mathematical. It is a matter of logical necessity for the child to develop this mathematical conception before he can use it for the next step, that of enumeration.

By enumeration, we mean using the number names to designate each successive object of a group which has been arranged in a serial order. In doing this, each object is recognized as an individual entity which should have a name of its own and a definite place in the series. Until rote counting is mastered, the child can not perform enumeration correctly.

When we have a collection of objects arranged in a series, we recognize that we can destroy the particular serial order by rearranging the objects. However, in doing this, we recognize that the group of objects has an aspect which is unchanged by the rearranging. The unchanged, or invariant, aspect of the rearranged series is the cardinal number of the objects. By identification in counting, we mean the determination of the ordinal number which serves to identify the cardinal number of the objects in the group.

In the above steps in counting, the child is first given groups of objects to study and enumerate. In the step of reproduction, the child is asked to perform the inverse and more difficult operation of forming the group of objects which corresponds to a designated cardinal number. The child does this by placing objects into a one to one correspondence with the ordered series of number names until he reaches the designated number.

In the step of comparison, two groups of objects are studied at once from both the ordinal and the cardinal points of view. For example, the child learns that a group of eight objects contains two more than a group of six objects. He also learns that one number stands between six and eight in the series of numbers.

By learning to count in the above senses, the child learns the names for numbers, and gains a clear understanding of the decimal system of notation, and of the meaning of number in both its ordinal and cardinal senses. Since both of these meanings are much used socially, he makes an important start toward understanding how adults use the number concept.

Counting also forms the essential foundation for learning the meanings of addition, subtraction, multiplication and division, all of which in the final analysis can be reduced to counting. A knowledge of counting enables the child to develop for himself the elementary facts used in our rapid methods for performing these operations.

The numeral frame was developed as a teaching device to use in every step in counting for demonstrating to the child, and in having the child discover the essential ideas in counting, and the meanings of addition, subtraction, multiplication, and division as developments from counting.

Rote counting, that is, learning the names for numbers in their serial order, may be taught by having the child move the blocks along the rails one at a time, and say the number names without placing any emphasis upon the serial position of each number. In this activity, the kinesthetic sense helps in establishing a cor-

* Herbert F. Spitzer, *The Teaching of Arithmetic*, Chapter III. (Chicago, Houghton Mifflin Co., 1948.)

respondence of the number names of the blocks.

Teaching enumeration or ordered counting, the next stage in the counting process, is accomplished with the numeral frame by having the child move the blocks along the rails and designating each block with its particular name. The arrangement of the blocks along the rails serves to emphasize their serial order. Having the number symbols on the blocks assists the child in forming the association of the numerals with the serial concept of number and with the number names.

The meaning of cardinal number, and its relation to ordinal number, which is termed identification, is taught by having the child move blocks one at a time to form a group. In doing this, the child says the name of each number as he moves the block into the group, and the last number named is the cardinal number of the group. Since number symbols are placed upon only one side of the blocks, and since the blocks can be turned around on the rails, the symbols may be, or may not be visible to the child. Both arrangements have advantages.

Reproduction of a designated number is accomplished by moving the desired number of blocks into a group, and the symbol upon the final block serves the purpose of verifying the child's counting.

In teaching the comparison of groups, the teacher shows the child how to form two different groups by placing the blocks into two groups upon the rails. Then the teacher shows how to compare the groups by adding blocks to a group and by removing blocks from a group. The teacher also shows the child how to compare the position of a number in the series of numbers. For example, if the group contains five blocks, the child recognizes that its number is two less than seven and three more than two.

The nature of our decimal system of notation is indicated in the numeral frame by having ten rails, and ten blocks for each rail. In this arrangement, as shown by Figure 1, numerals having the same final symbol form a column. The object of this arrangement is to make it easy for the child to see and understand the systematic character of our system of notation.

The numeral frame is also useful in exercises in which counting is used to obtain the sum, difference, product, and quotient of two numbers. The child, having mastered the step of reproduction, can produce groups of any designated number. Then he can put two groups together, and by counting determine the sum of the two. Similarly, he may form an arbitrarily designated group, and then remove from it a designated number, and in this manner determine the difference between two numbers. It is evident that since equal groups can be put together, the meaning of multiplication can be demonstrated easily. Starting with a number such as twenty-five, the teacher may show the child how to divide the blocks into five equal groups, and in this manner show the meaning of division.

From the above discussion, it will be seen that the numeral frame serves as a powerful means used by the teacher for communicating ideas about number to the mind of the child so that these ideas become perfectly clear and firmly fixed. The numeral frame also enables the child, through his handling of the blocks, to communicate his thoughts to his teacher, who can then judge what he has learned and what he has failed to learn. Therefore there is a closer meeting of minds than we can obtain without such a device.

The teacher who starts to use this device will quickly find that one of its important contributions to teaching is that it makes it easy to obtain and hold the close attention of an entire class. Moreover, children greatly enjoy using this device, and their joy motivates and gives purpose to their learning.

Secondary School Mathematics in Great Britain and the United States*

By William S. Brace

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1. Privilege of Speaking

I am very conscious of the honor done to me and to the teachers of Britain by your president in asking me to address you. My qualifications for talking about American schools and educational methods are very slight indeed compared with those of many of you here today, so I am quite sure that it is not on those grounds that I am being asked to speak to you. Perhaps it is that someone is a little suspicious of the report that I am going to carry back to England, and is contemplating "bumping me off" if I haven't formed satisfactory impressions of this beautiful state of yours! For my own safety, then, I will say at once that I think that Colorado is one of the most beautiful places in the world, that the people of Colorado are among the most hospitable in the world, and that the schools of Denver, which are the only Colorado schools that I know much about, are among the best in the world.

2. Greetings from Sheffield

Shortly before I left England I attended a meeting of the Sheffield Branch of the Mathematical Association of Great Britain, an organization corresponding to the National Council of Teachers of Mathematics in the United States, and I was commissioned to bear their greetings to the mathematics teachers of the part of America that I was to visit. I have very great pleasure at this time in delivering that message of greeting.

* Presented at a meeting of the Colorado Council of Teachers of Mathematics, April 20, 1951.

3. Personal Attitude in This Address

I feel that this address to some extent puts me "on the spot," for one of the duties of an exchange teacher is to try to further friendly relations between the two countries, and it has been well said that "comparisions are odious." It seems that there are three courses open to me: first, to say that I find everything in the United States is wonderful, and be convicted before all of you here who know better as being an ignoramus or a sycophant; second, of allowing my critical propensities full reign, and finding everything which is not British absolutely horrid; or, third, to try to steer a middle course and give an objective analysis and assessment of the differing arrangements of the two countries. It is obvious that one of these courses would serve no useful purpose. For some audiences the first approach might be desirable, but I am sure that this audience is of stronger fiber, so I shall adopt the third method, and endeavor to give an honest account of my experiences.

4. Analysis of the Secondary School Systems of Britain and the United States

The differences in organization between the schools of Britain and those of the United States are great enough to affect very appreciably the courses and methods of teaching in the two countries, so I had better begin by outlining the major differences in this area. At the secondary level in the United States you have a system of common schools. The goals of these schools are, according to "Planning for

American Youth,"[1] largely social, but there are also aims which seem to have a bearing in the field of mathematics. The imperative needs of youth which can be at least partially satisfied by teachers of mathematics in their mathematics classes include the development of salable skills and those understandings and attitudes that make the worker an intelligent and productive participant in economic life: the development of knowledge of how to purchase and use goods and services, understanding both the value received by the consumer and the economic consequences of their acts; the development of an understanding of the methods of science and the influence of science on human life; the development of the ability to think rationally, to express thoughts clearly, and to read and listen with understanding. No one pretends that all these needs are satisfied exclusively, or even primarily through the mathematics classes, but it does seem that these needs provide the justification for the inclusion of a considerable amount of mathematical work in the programs of all pupils.

In Britain general aims are less well formulated. Our broad aim is to give each child an education suited to his age, aptitude and ability, without enquiring too closely just what fulfills these conditions. We break our secondary schools up into three parallel streams, each covering the whole secondary period, instead of into two consecutive phases, as is common here. Our three streams cater, broadly, for those who require a college preparation, in rather a narrower sense than is usual here, for those who require a thorough preparatory training so that they may become technicians or craftsmen, and for the general run of workers. Access to the three streams is controlled solely by ability, and social or economic status has no direct bearing. In addition we have a rather well developed system of private schools, known for historical reasons as public schools. These have, I judge, rather more impact on the community at large than comparable American private schools.

[1] National Association for Secondary-School Principals, *Planning for American Youth.* (Washington, D. C.: National Association for Secondary-School Principals, 1944.)

5. My Experience

I feel that it is only fair that I should make it clear at this stage that my experience in Britain is almost exclusively with the college preparatory stream, and quite largely with the best students of mathematics in that stream. This makes comparison with the general run of pupils in the American high school rather difficult for me. Out of consideration for my limited experience, my colleagues on the faculty at South High School have seen to it that the greater part of my work here has been with those pupils showing some evidence of mathematical ability. The most elementary classes I have had to teach have been an Algebra 1 and an Algebra 2 class. I have been into one or two junior high schools and observed a few classes there. But by some mischance when I visit a school, instead of being given a chance to learn, I am looked upon as some peculiar performing specimen, and called upon to give lectures on all sorts of remote and recondite subjects! I trust that I have made it clear that I cannot speak with any sort of authority about any type of school, English or American, except the English grammar school.

6. Preconceptions

I arrived in America with certain preconceptions about what I should find in American high schools. Those preconceptions, in so far as the teaching of mathematics was concerned, were based largely on The Mathematics Teacher, which I have read for some years, and also on the wonderful year-books put out by the National Council. I had been greatly impressed by the tremendous amount of research, painstaking and detailed, which is being done into the technique of teaching mathematics, and which I had seen re-

ported in those documents. I had also read a good deal about the work being done to humanize mathematics, so that the pupils might have a more adequate motivation, so that they might see the relevance of the work they were doing to the experiences they met in their daily lives, and so that they might understand the basic mathematical concepts. Education in England is in a state of abnormal fluidity at the present time, and one of the reasons why I applied for an exchange with an American teacher was so that I might see how these new ideas and some of this research was being applied. My reactions to what I have seen here are naturally colored by my background of experience and by the success I have had in looking for the things I hoped to find.

7. Mathematics Teaching in the United States

As I now begin to comment on what I have found in the American high schools I must again emphasize that what I say is based on what I have seen in the high schools of Denver and in a few of the school districts of Colorado. I am well aware that this is not an adequate sample of the schools of the United States. The mathematics classes which I have found to be common to all the schools with which I have come into contact seem to me to fall into three groups. First there are the so-called remedial mathematics classes, going under a variety of different names, but all having as their objective the removal of certain defects in basic mathematical competence or understanding. It would seem that there are two possible ways of dealing with these difficulties: one is to work on the assumption that the previous teachers of the pupils were incompetent or negligent and did not give the pupils sufficient practice; the other is to conclude that the pupils were presented with certain material when they had not sufficient maturity for that material to be meaningful to them. The former attitude might be called the classical attitude; the second appears to be the one taken in the research articles in The Mathematics Teacher. I have been impressed by the excellent results being achieved in many of these classes, by the care taken to identify the exact skills lacking, and the trouble taken to remedy defects identified. But I have not yet observed a class being conducted on other than classical lines, and I have spoken to only a very few teachers who made it clear to me that they were endeavouring to use the results of recent research.

The second type of class which I have met is the general mathematics class. These classes, it is said, mark an attempt to get away from the sub-division of mathematics into watertight compartments, at the same time making the work relevant to modern life and meaningful to the pupils. In practice the classes frequently turn out to be arithmetic, with a very slight admixture of algebra—substitution in easy formulas and a few simple graphs, perhaps with the intention of introducing the idea of functionality—and a little geometry—some mensuration. It is stated that these classes are for all pupils, but in practice it seems that the weaker pupils are the ones who find their way into them. Again I would say that an excellent job is being done within the limits accepted, and the teachers strive manfully to overcome the limitations of the situation in which they find themselves. Nevertheless, the classes in practice seem to me to be rather different from what was envisaged by their proponents.

The third type of class is the traditional algebra, geometry, solid geometry, trigonometry, or mathematical analysis class. These classes move on solid traditional lines, and concentrate on giving a thorough training in the basic techniques of the branches of mathematics being studied. In the algebra classes particularly, the subdivision of the subject matter is not quite so complete as the titles of the classes seem to indicate, and some work of a geometrical type is often included. Standards

attained seem to be reasonably high, having due regard to the amount of time spent and the age at which the studies are started. The most interesting classes with which I have come into contact were a complete surprise to me—classes ostensibly in plane geometry, but actually having as their main object the exploration of the theory and practice of clear thinking. These classes are obviously an attempt to apply the psychological discoveries of some thirty years ago to the announced aims of formal geometric teaching. The work I have seen being done is of a very high standard indeed. The pupils really seem to gain not only a good understanding of logical processes, but also considerable powers of critical thought, so that they are much less easily fooled by plausible but spurious arguments. For some curious reason, although harder critical and constructive thinking is called for than in the traditional geometry courses, the weaker pupils are the ones generally counselled into these courses. Even so, the results achieved are more than satisfactory. This is most certainly an idea which I propose to take back to England with me.

8. Mathematics Teaching in Britain

Mathematics teaching in England is organized rather differently. In the first place most of our pupils enter a secondary school when they are between eleven and twelve years old—say as they enter the seventh grade—and remain in that school until their school days are over. There is a wide variation in the way mathematics courses are arranged within a common framework which is quite generally adopted. The division of mathematics into separate subjects so that for one term a pupil will do algebra exclusively, for another term geometry exclusively, and so on is not favored. I would like to say that this is because we regard mathematics as a single subject, and that we adopt any of its tools as they seem to be appropriate for the subject in hand. That idea is preached by the leaders of thought in mathematics teaching. The real reason, I fear, is the tradition that all pupils shall start some geometry and some algebra in the seventh grade. In practice this means that if five periods a week are allocated to mathematics—we work on a weekly timetable, rather than a daily timetable—then two periods will be allocated by the teacher to arithmetic, two to algebra, and one to geometry, and the subdivision is as complete as it is here, except that the different branches are taught concurrently, and usually by the same teacher. Some teachers really do strive for integration of the different branches, and I think that the number doing so is increasing very steadily. Such teachers look upon the study of mathematics primarily as the study of number, space, and time, particularly of the ways in which these enter modern life.

Our courses in mathematics are planned as four or five year courses in which the basic concepts are encountered. We try to help our pupils to acquire those basic skills which they need while at school, and which they will use as young adults. We also endeavour, by simple and vivid examples, to bring them into contact with the great ideas of mathematics and their applications to the life of our civilization. These aims are high aims, and are certainly not often satisfactorily achieved, but I do think that during the last twenty or thirty years, in pursuit of these aims, we have gone a long way in cutting out dead wood that has obstructed mathematical teaching and made the name "mathematician" synonymous with "unworldly crank." We have, I think, gone a good deal further than you have in removing manipulation for manipulation's sake and juggling for juggling's sake from our elementary mathematics. In consequence, we have more time to stress the really important ideas. The pupils who need the manipulative powers have the necessary background of ideas on which to build when the time comes, and then, because they have a real reason to do the

manipulation, they are sufficiently alert to get it right. Prompted by a personal sense of urgency they quickly develop the necessary skills.

The five year course includes some of the arithmetic of finance, algebra of simultaneous quadratic equations and finite geometric series, geometry to the properties of circles and of similar polygons, a little trigonometry, say the use of the six ratios, and some very elementary analysis. This course is started by almost all pupils, though not all pupils go the whole way. We go in rather thoroughly for homogeneous grouping in our mathematics classes, which helps us to temper the wind to the shorn lambs, or to cut our coat according to our pupil's cloth.

The greatest difference in teaching methods I have found has been in the field of plane geometry. Here the ideas of geometry and the use of mathematical instruments are introduced on a limited scale in the seventh and eighth grades. Then there is a gap while algebra is taken up, so that not until the tenth or eleventh grades is formal geometry begun. The few pupils who remember anything about their geometrical instruments after this interval are then made aware of the sin of using those instruments. I have found in my own classes pupils saying that if triangles are congruent, corresponding angles are equal—and not knowing how to measure an angle to test the accuracy of a congruence construction. Faith in reasoning is good, but a realization of the frailty of human reasoning is also scientific.

In the early twenties the Mathematical Association of Great Britain set up a committee to examine and report on the teaching of geometry. This committee suggested a three stage approach: Stage A—gaining experience of the basic spatial ideas by drawing and measurement; Stage B—ad hoc proofs of properties discovered in Stage A, making such assumptions as appeared reasonable; Stage C—establishment of a logical geometric structure. These stages were visualized as consecutive but overlapping, with gradually increasing emphasis on stages B and C. In practice, Stage A is the only type of geometry considered for about a year (one or two periods a week), and much emphasis is placed on accurate and correct use of the standard geometrical instruments—ruler, compasses, dividers, protractor, triangles. After the first year the five year course will be a mixture of the first two stages, with stage B progressively displacing stage A. For the most able pupils some attention may be paid to stage C in the last year. This approach gained rapid acceptance, and is still being developed. It is clear, of course, that the subject matter is primarily geometry, and only secondarily, reasoning processes.

When pupils have completed this five year course successfully—success being determined by performance in an examination—they have fulfilled the minimum requirements for college entrance. They will not proceed to college for a further two or three years, so those who propose to study mathematics at the university will begin a further course extending over those two or three years. The normal time allowance will be eight to twelve periods weekly, and in this course a considerable amount of work in analysis, trigonometry, solid and analytical geometry, and theoretical mechanics is done. This special group of able pupils do work which is considerably more advanced than anything which is done in high schools here.

9. Textbooks

The teaching of mathematics, and indeed of most subjects, is for the average teacher, closely bound up with the textbooks he has to use, or which are available to him. The work of gathering suitable illustrative material and of compiling a sufficiently lengthy and closely graded list of practice exercises is so great that the teacher with a full program can hardly contemplate it. He certainly cannot do it

and at the same time keep up with the necessary correction and grading of papers. Naturally he relies on the textbook for much of his illustrative material and almost all of the drill examples. This means that the teacher's methods and approach are very much affected by the attitude of the textbook writers. Now it is exceedingly unfortunate that the large amount of research which goes into the production of a new textbook, and the capital investment which it represents imply that every textbook must be sufficiently conservative in outlook to command a wide sale, and radical experiments with textbooks which from their nature are likely to need considerable revision as experience accumulates are almost prohibited. American textbooks on conservative lines and which concentrate on carefully arranged and graduated drill are very good indeed. The arrangements made for a pupil to assess his own progress and identify for himself the work he needs to do excite my intense admiration, for I have not met books of this type before. Many English teachers would be quite shocked at the idea that a child could possibly discover his own weaknesses, and that, having discovered them, he would have the fortitude to work at them.

On the other hand, the more advanced textbooks have been rather disappointing to me. Quite frankly, most of the mathematics textbooks now in use in the Denver public schools would be regarded in England as at least twenty years out of date, and that by those enemies of change, English teachers. The features which lead me to this conclusion are, in algebra, the emphasis on drill and formal manipulation, often of a very complex type, at the expense of reasonably attractive story problems and other applications of the algebraic methods considered, and, in geometry, the tendency to start very quickly with formal axioms and postulates which are presented as such, and which often relate to entities with which the pupils have had little or no first-hand contact. I am bound to say, however, that some of the most recent textbooks appear to me to show a very considerable advance.

In Britain we seem to have a much wider variety of textbooks available, including many with a far less formal approach. I may have this impression because my knowledge of English textbooks has been built up over a number of years, and I now know almost exactly where to look for a text of any particular type. One rather exciting book which has been introduced lately and is intended for the less mathematically able pupils has the title *Designing and Making.** It is a collection of about one thousand experiments of a type likely to appeal to boys and girls who can use their hands, and illustrating or involving the basic mathematical ideas which we are all trying to get across. This is, admittedly, a radical departure, but it is based on the experience of the author, who is a practicing teacher, until recently at the secondary level. It has certainly aroused a great deal of interest in Britain. On more orthodox lines we have many excellent textbooks giving a wide variety of applications of the various mathematical ideas and concepts with which they deal. There are also more formal books, some of which have been in use for forty years and are being replaced by the more modern books as rapidly as school finances will permit.

As an example of the sort of differences which I am referring to, most American algebras I have seen start with a short chapter on the use of formulas, and then get down to a lengthy study of the use of letters for numbers and drill on the basic operations with letters. The English method would be to make the chapter on formulas more substantial, and include in it much of the formal work on the four rules as incidental to the manipulation of the formulas; alternatively, many English algebras start with the solution of simple

* W. W. Sawyer and L. G. Srawley, *Designing and Making.* (Oxford: Basil Blackwell, 1950.)

concrete problems, introducing algebra in the historical way through the syncopation of verbal statements. With either method there is an understandable reason for the drill work involved. Another example which struck me forcibly: a solid geometry text which I was given to use has some beautiful pictures of airplanes, buildings, machinery, etc. and the student is informed that solid geometry is used in the design and making of these things, but I have searched the book in vain for some reference hinting what or how. An English text would probably be short on the pictures, but would have some real information in the text.

It seems to me that the function of the textbook is not well understood either in Britain or America by the writers of these necessary instruments. Some textbooks are apparently designed to make a teacher unnecessary; in these books there is much waste of space on detailed instructions better given by the teacher. Other books are just collections of examples, leaving all explanations and applications to the teacher. I consider that any satisfactory textbook should perform three functions: first, it should provide a lengthy list of practical applications of the matter under consideration—the average teacher does not have the knowledge, the facilities, or the time to compile this information each day; second, the textbook should provide a comprehensive set of well graded examples and drill material—again the average teacher has neither the technical skill nor the time to prepare such a list; third, the textbook should provide for the pupils' reference a simple explanation of the common procedures. In my experience the first function is grossly neglected in favor of the second. The teacher is thus left without the assistance which he could reasonably expect.

10. Organization and Uniformity

Before I conclude I feel I ought to say something about the way in which a measure of uniformity between the schools is achieved in the two countries. In both countries the schools are free from central control of curriculum. In England each school, in theory, can make its own syllabuses and curriculum, according to its own ideas; in America each school system has the same liberty. Yet no one could possibly mistake an English scheme of studies for an American one, or vice versa. Why? Whence the uniformity in view of the liberty to vary? In America there is great uniformity within a school system, and for good reasons; a measure of uniformity between school systems is secured partly by the agreement there is between college requirements, partly by the popularity of the various attainment tests. For better or for worse many teachers at the high school level seem to have at least part of an eye on one or other of these standardizing influences. In Britain uniformity at the elementary level within a school system is brought about by the Selective Examinations for Entrance to Secondary Schools. Each school system organizes its own examinations, and it is essential that the pupils shall be adequately prepared for the examination they will take. Uniformity within the secondary schools is brought about by the General Certificate of Education Examinations, which are effectively college entrance examinations. At each level there is a systematic arrangement for consultations between teachers and examiners.

In both countries, therefore, the colleges have an important unifying influence and the differences between the courses in the schools of the two countries are to a considerable extent due to the differences in the outlooks of the universities. In Britain, universities are regarded as places of academic learning, selecting their pupils accordingly. In the United States every person who wishes to lay claim to an education must have attended a university, and demands the right to attend, so the universities modify their courses accordingly. Of course in each country the

(*Continued on page 394*)

The Slide Rule Made Meaningful

By George L. Keppers *and* Perry A. Chapdelaine
Iowa State Teachers College, Cedar Falls, Iowa

What constitutes "meaning" in slide rule instruction? The most frequent answer, probably, is that the student should know the main principle underlying slide rule construction. That principle, of course, is the representation of the logarithm of a number by a distance along a line as the decimal part of a given constant.

It has been within the experience of the writers to note the results of faulty slide rule instruction in their classes. On questioning the pupils, it would seem that teachers follow one general procedure to effect slide rule competence. The teacher may have a large wall model or, most generally, just a ten inch slide rule and a manual "to accompany the slide rule." The students are equipped with a slide rule. The various scales are explained and there may have been some preliminary work such as a thorough study of logarithms (a thorough study is not necessary) and a study of scientific notation. Next the teacher suggests that a simple problem be worked such as $2\times 3=6$. The procedure is as follows: line up the left hand index of the "C" scale with the two on the "D" scale, then move the hair line to the three on the "C" scale and read the answer on the "D" scale. This is continued for many other examples including problems in division, finding square roots and raising to a power. By this method the student can, in many cases, become proficient at manipulating the slide rule. However, in most cases the student does not know what he is doing or why he is doing it.

The writers would propose an exercise for the students to perform which has worked successfully in their trigonometry classes. Remarks such as these: "How interesting!," "Well! I never realized it was this simple!," "Why wasn't I taught this before?," and so on, are evidence that the procedure recommended is meaningful and simple to perform.

In teaching the slide rule in a meaningful way, a certain amount of preliminary work is necessary. The student should know and understand the four fundamental operations of logarithms—multiplication, division, raising to a power, and finding a root. More specifically, if the student is given $\log_{10} 2=0.301$, $\log_{10} 3=0.477$, and $\log_{10} 7=0.845$ and knows that $\log_{10} 10=1$, he should be able to compute and understand such problems as: $\log_{10} 14=x$; $\log_{10} 2=x$; $\log_{10} 7000=x$; $\log_{10} 21=x$. It is not necessary that the student be thoroughly familiar with scientific notation but such a familiarity can aid in the learning process.

The necessary equipment for performing this exercise consists of a ruler, marked either in centimeters or tenths of an inch (this article is based on the centimeter system), ordinary note book paper having a diagonal at least thirty centimeters long, a table of logarithms, a pair of scissors, and a pencil. A slide rule is not recommended.

First, each student should draw a straight line thirty centimeters long, diagonally across his paper as shown in Figure 1.

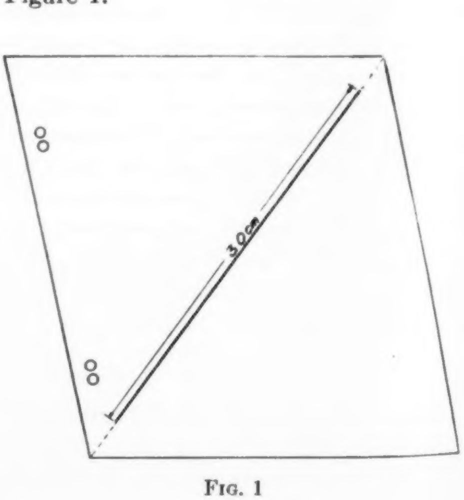


Fig. 1

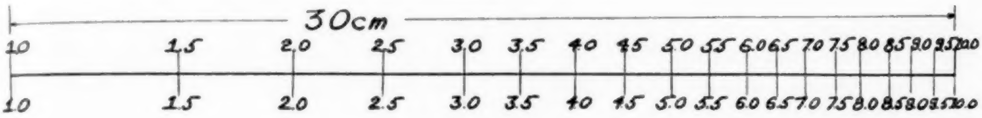


FIG. 2

Next, have the students find the logarithms of 1, 1.5, 2, 2.5, 3, . . . , 9.5, 10; multiply each of these values by thirty and round off the products to the nearest one-hundredth. The class as a whole can participate in finding these values while the teacher records them on the blackboard. If the students have not used the tables previously, the teacher may have to give a short explanation in finding these logarithms. A more detailed slide rule can be made by using more divisions but it is advisable to limit the number of computations to avoid confusion. As each of the products is formed a mark is made along the diagonal to designate centimeters from the end of the diagonal. The marks are numbered as illustrated in Figure 2.

The instructor can sketch the procedure on the blackboard to facilitate his explanations.

The number one is placed above and below the mark at the end of the line because the logarithm of one is zero and would be represented by zero centimeters. The number two is placed above and below the marker which is approximately equal in length to 9.03 centimeters. This value is obtained by multiplying $\log_{10} 2 = 0.301$ by thirty. In this manner we are representing the logarithm of a number by a distance along a line as the decimal part of thirty centimeters. These two sets of numbers above and below the line are referred to as the "C" and "D" scales, respectively.

After the above mentioned values have been located, the paper should be cut along the line separating the C and D scales. Now the student is ready to apply the rules for multiplying and dividing with logarithms. By solving simple problems such as $2\times3=6$ and $5/2=2.5$ the student can readily see that when using the slide rule he is doing nothing more

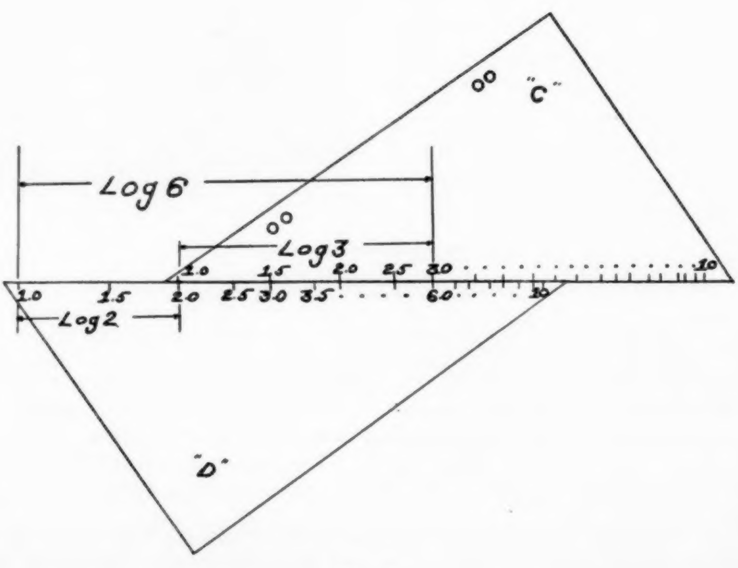


FIG. 3

than adding or subtracting lengths which represent logarithms as shown in Figure 3.

If one wished, he could emphasize the fact that a logarithmic scale is not equivalent to a linear scale by having the students construct a linear scale "slide rule" and noting the products on this scale.

More difficult problems should be introduced as the students become familiar with these two scales. The teacher may ask, "How would you construct a scale to find square roots?" or "How would you construct a scale to find the twelfth power of numbers?". The scales for finding square roots and squaring numbers can be developed by applying the rules used in the work with logarithms. That is, the students could make one scale half the size of the other since we multiply the logarithm of a number by two in squaring.

Mathematics will not be the practical or cultural tool necessary for the development of "correct" understandings until the mysticism of mathematics can be explained in terms of the pupils' experiences. After pupils have had experiences of the kind proposed above, the writers feel that students will find working problems with the slide rule not only an expedient procedure in arriving at an answer but also a fascinating and meaningful exercise.

Secondary School Mathematics

(*Continued from page 391*)

schools have a very considerable influence on the universities, but certainly in England the universities have a very big influence on the schools. Further, both schools and universities reflect the social and economic philosophies of the countries.

11. General Impression

I suppose that I would say that my general impression of mathematics teaching in America is one of disappointment; disappointment that the teaching is considerably less progressive than I had expected. But then as I reflect, this sense of disappointment passes—I was looking for the moon, and naturally I did not find it. Wonderful work is being done; research is being pushed further and further into the best ways of learning and the best ways of teaching. What if this research is a little slow in gaining acceptance? Education is no field for revolutionary changes—the lives of people are at stake. Gradual progress must be our aim. Before we cast off the old we must be sure that the new is better.

One thing I certainly have learned here —that it is possible to make social aims the center of an educational program, that it is possible to make a success of education for citizenship. Perhaps the day is not too distant when education for world citizenship will be the center of education in every country of the world!

REVISION OF BY-LAWS

Revisions of the By-Laws of the National Council of Teachers of Mathematics as printed in the February, 1951 issue of The Mathematics Teacher were adopted at the annual business meeting of the National Council in March. The president, however, asked the committee which had worked on these revisions to continue another year to consider any further changes which might be proposed by the membership. Announcement that the committee would receive these suggestions was made at the annual business meeting and has been sent to all affiliated groups. Suggestions can still be considered for action at the next annual business meeting if received by the committee before November 20.

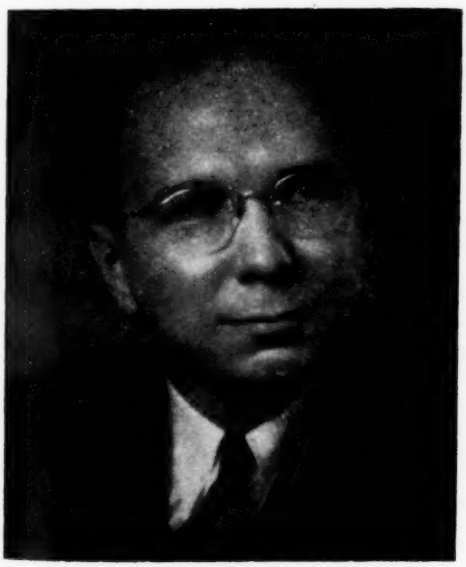
Members of the committee are George Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois; Mary Potter, Board of Education, Racine, Wisconsin; and Marie S. Wilcox, Chairman, George Washington High School, Indianapolis, Indiana.

The President's Page

MEET OUR EXECUTIVE SECRETARY

LAST May the Board of Directors of the National Council of Teachers of Mathematics appointed Mr. M. H. Ahrendt as our new Executive Secretary. Mr. Ahrendt assumed his duties at our headquarters office at 1201 Sixteenth Street, N. W., Washington 6, D. C., on July 1, 1951. Prior to this date, this position had been filled temporarily by your President, during our first year as a Department of the National Education Association.

Mr. Ahrendt has been an active member of the National Council for several years. Many will remember the fine work he did as General Chairman of our annual meeting in Indianapolis in 1948. We all know him through his good work as Chairman of the Mathematics Kits Committee.



M. H. AHRENDT

Mr. Ahrendt comes to us from Anderson, Indiana, where he has been for the past four years teaching mathematics and physics at Anderson College. He has had many years of teaching experience in junior and senior high schools in Indiana and has done some work in teacher training and in supervision of practice teaching. Mr. Ahrendt was President of the Indiana Council of Teachers of Mathematics in 1947–48 and was Secretary of the Mathematics Section of the Indiana State Teachers Association in 1949–50. He was Chairman of the Reorganization Committee of the Mathematics Section of the Indiana State Teachers Association in 1949. This committee developed a plan for consolidating the various organizations of mathematics teachers in Indiana. The plan was adopted and is now in operation.

In 1932 Mr. Ahrendt received his M.A. degree at the Univeristy of Wichita, Wichita, Kansas. He did additional graduate work at the University of Cincinnati in 1933–35, at Ball State Teachers College in 1937–39, and at the University of Michigan in 1948. He has done much independent study of advanced and enrichment topics in mathematics, one example of which is his work with "linkages." He has written several articles on mathematics, most of which are to be found in THE MATHEMATICS TEACHER. His hobbies include woodworking, photography, and music. He is married and has a 15-year-old daughter.

Mr. Ahrendt comes to us with high recommendations. He has shown himself to be honest, conscientious, and a hard worker. He has demonstrated originality and initiative. Let's give our loyal help and cooperation to Mr. Ahrendt that he may discharge his duties well, that he may carry on successfully the old services to our members and help to develop new ones, and that he may increase our cooperation with other organizations for the betterment of mathematics teaching.

We wish him happiness and success.

H. W. CHARLESWORTH,
President

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

30. A Second Note on the Pythagorean Theorem[1]

The purpose of this note is to give a proof of the converse of the Pythagorean theorem which is independent of the theorem itself and which uses only theorems of the first book of Euclid which would have been known to the ancient geometers.

CONVERSE OF THE PYTHAGOREAN THEOREM: *If the square of the number which measures one side of a triangle equals the sum of the squares of the numbers which measure the others, the figure is a right triangle.*

Let ABC (Figure 1) be a triangle the measures of whose sides BC, CA, and AB are respectively a, b, c which numbers satisfy the relation

(1) $$a^2+b^2=c^2.$$

(2) Then $a^2=c^2-b^2=(c-b)(c+b)$.

On BC, AB and BA extended locate points B_1, A_1, D, E, such that $BB_1=c-b$, $BA_1=BC=a$, $AD=-AE=CA=b$. Hence $BE=BA+AE=c+CA=c+b$ from which, using (2), we have

(3) $$\frac{BB_1}{BC}=\frac{c-b}{a}=\frac{a}{c+b}=\frac{BA_1}{BE}$$

which establishes the parallelism of A_1B_1 and EC. From this it follows that angles $B_1\,A_1\,B$ and CEA are equal.

Triangle $A_1\,BB_1$ is congruent to CBD since they have a common angle B comprehended between equal sides. Further triangle CAE was constructed to be isosceles, hence

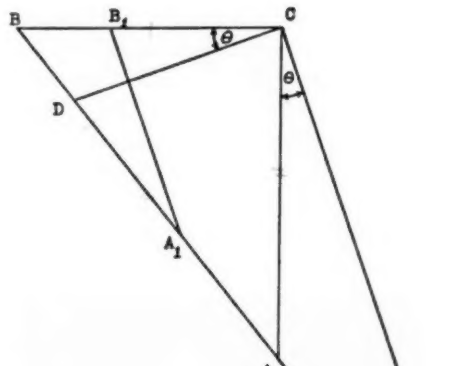


FIG. 1

$$\angle BCD=\angle B_1A_1B=\angle CEA=\angle ECA=\angle\theta.$$

This last shows that $\angle ECD$ if rotated through angle θ would coincide with $\angle ACB$. But $\angle ECD$ can easily be proved to be a right angle since $CA=DA=AE$, or, in other words, the median to DE is $\frac{1}{2}\cdot DE$, and hence, finally, $\angle ACB$ is a right angle which was to be proved.

Victor Thébault,
Tennie (Sarthe) France

31. An Approach to Determinants[2]

Two of the reasons for the difficulties encountered in teaching determinants

[1] See THE MATHEMATICS TEACHER. Vol. XLIII (Oct., 1950), p. 278 for the first note.

[2] The ideas for this note can be traced to Felix Klein, *Elementary Mathematics from an Advanced Standpoint—Geometry*. (New York: Macmillan, 1939), Part I, Section I; T. H.

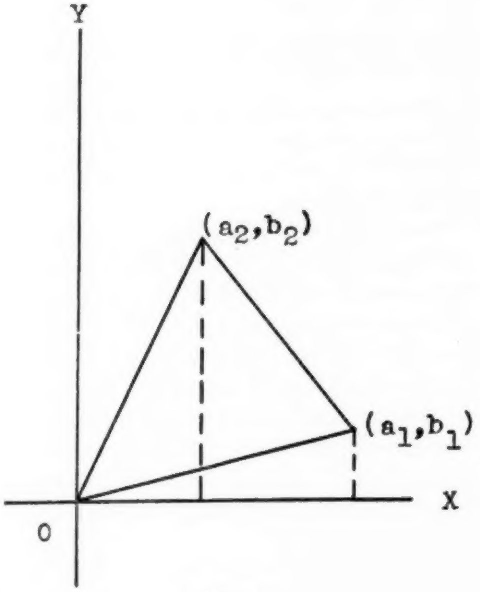


FIG. 2

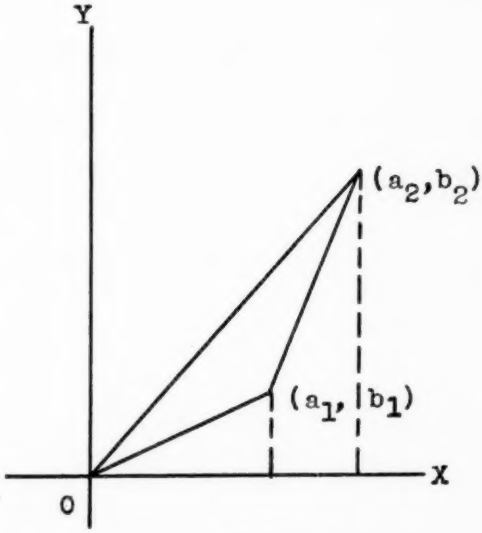


FIG. 3

are (1) the difficulty of motivating their introduction so as to make them appear other than arbitrary, vague, "plucked from thin air," and (2) the reliance for concrete application almost solely upon Cramer's rule which does not seem too important to students since they have other reliable and simple procedures for simultaneous solution of equations.

The following parallel development of algebraic and geometric approaches to determinants partially overcomes these difficulties and even provides some motivation for the derivation of the properties of determinants.

The work begins by suggesting that, as with the quadratic so with simultaneous linear equations, a formula for their solution might be a worthwhile tool if it were simple enough and the problem were to occur often enough. It is easy to derive for $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ that

$$x=\frac{c_1b_2-c_2b_1}{a_1b_2-a_2b_1}$$

and a similar formula for y. At this stage the second order determinant may be introduced as a mnemonic device for recalling this formula.

It may then be suggested that geometric interpretations and applications are often illuminating and that since in each determinant there were two pairs of numbers we might consider a new geometric problem in which we have two points which joined to each other and to the origin form a triangle. If, starting from the origin, we walk with the area of the triangle on our left, and use (a_1, b_1) to represent the first and (a_2, b_2) the second point to which we come, one of figures 2 or 3 must result (limiting ourselves to the first quadrant for convenience). The area in both cases is then easily seen to be $A=(1/2)\ (a_1b_2-a_2b_1)$. For example, in figure 2,

$$\begin{aligned} A&=(1/2)a_2b_2+(1/2)(a_1-a_2)(b_1+b_2)\\ &\quad-(1/2)a_1b_1\\ &=(1/2)(a_1b_2-a_2b_1). \end{aligned}$$

It takes only a moment for students to recognize this as of the same form found in the solution of our simultaneous equations. They now have two quantities representable by determinants and will be quite ready to agree that there may be

Hildebrandt, "Marginal Notes." *The American Mathematical Monthly*, XXXVI (April, 1929), p. 216 ff. and conversations with various persons, especially Norman Anning and P. S. Dwyer.

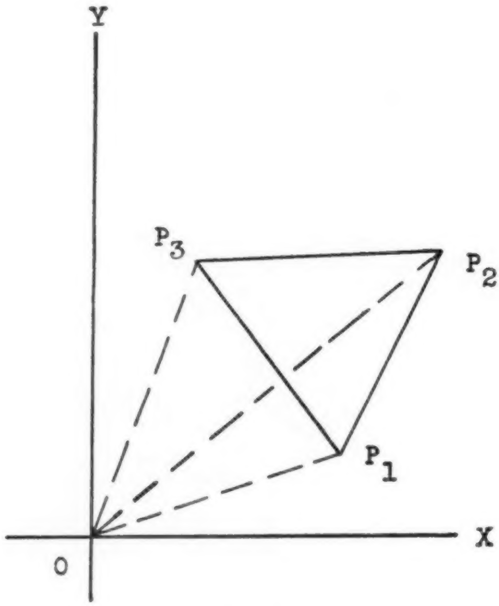


FIG. 4

value in a study of the properties of such arrays.

Next we observe that if the points are reversed in their order of appearance in the area formula then

$$A' = \begin{vmatrix} a_2 & b_2 \\ a_1 & b_1 \end{vmatrix} = a_2b_1 - a_1b_2 = -A.$$

We associate this negative area with walking around the triangle clockwise. Returning to the solution of simultaneous equations we note that this exchange of rows would, formally, correspond to interchanging the order of the equations, and would quite properly not change the solution merely the signs of both numerator and denominator. We now have derived and examined the implications in two situations of the rule about interchanging rows. The result for columns of the second order determinant can be easily derived.

From here we return to the solution of simultaneous equations in three unknowns, obtaining by straightforward elimination

$$x = \frac{d_1(b_2c_3 - b_3c_2) - d_2(b_1c_3 - b_3c_1) + d_3(b_1c_2 - b_2c_1)}{a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)}.$$

The students will recognize the determinants here and the instructor may introduce the third order determinant as a mnemonic for this expression which of course is equivalent to using the expansion by minors as a definition of the third order determinant.

Naturally we next seek a geometric counterpart. Two are available, extension from area to volume or from two to three arbitrary points in the plane. The latter is simpler. Consider the triangle none of whose vertices is at the origin (figure 4). Joining its vertices to the origin we see its area to be given by $OP_1P_2 + OP_2P_3 - OP_1P_3$ or

$$\frac{1}{2}\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$$
$$= \frac{1}{2}\left\{ 1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - 1 \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} + 1 \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right\}.$$

This expression is readily seen to be

$$\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

the well-known area formula.

Geometric thinking may then be used to suggest and give concreteness to such theorems as multiplication of a column by a constant, multiplies the determinant by the constant, and two identical columns or rows imply a zero value. Further applications of determinants to plane, solid, and analytic geometry follow naturally from such a beginning as does also their use in the construction of alignment charts.[3] A scheme used by civil engineers

(*Continued on page 400*)

[3] See, for example: Lawrence I. Hewes and Herbert L. Seward, *The Design of Diagrams for Engineering Formulas.* (New York: McGraw-Hill Book Co., Inc., 1923); Maurice Kraitchik, *Alignment Charts.* (New York: D. Van Nostrand Co., Inc., 1944.)

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
135–12 77th Avenue, Flushing 67, N. Y.

The purpose of a mathematical recreation when it is employed in the processes of mathematical instruction is many-fold. A mathematical recreation may be used in order to break the monotony of a classroom study. It may be wrapped in a humorous cloak. The appeal to the pupil's sense of humor may relieve the tension, may spark the waning interest in a specific topic.

A mathematical recreation may be used in order to motivate the development of a topic. Under such circumstances a mathematical recreation serves the purpose of creating the atmosphere which will be conducive to a fruitful and effective evolvement of a mathematical study.

A mathematical recreation may be used for the purpose of arousing the imagination of the pupils. Under such circumstances a mathematical recreation is expected to contribute to the growth of the mathematical experiences of the pupils.

Whatever the purpose of a mathematical recreation may be, its introduction in the classroom situation must consider several important factors which are continuously operative in the processes of mathematical instruction. It should be understood that the last statement implies that the operativeness of these factors may be either positive or negative. Of these two factors the most important are *curiosity* and *interest*. A mathematical recreation is expected to arouse curiosity as well as evoke continuous interest. If these two take place the operativeness is positive. On the other hand, when these two are absent, the effect is totally negative. However, the range between the two effects is such that there may take place some variations from the extremes. Under some circumstances the pupils may indicate a slightly mild interest, that is, they may respond to a mathematical recreation as if it were "just another problem."

In other words, the employment of mathematical recreations as means of instruction has its pitfalls against which the teacher must be always on guard. Justly, a mathematical recreation is an instructional instrument more dangerous than any other means of instruction. The interest of a teacher, the enthusiasm of a teacher is generally not the best criterion for the judgment of the usefulness of a mathematical recreation in a classroom. Furthermore, the standardized and classical types of mathematical recreations, such as (a) the guessing of a number, (b) the guessing of date or of an age, or of a day of a week, and many other mathematical recreations are, in themselves, dull and dry mathematical exercises, replete with algebraic relations which are complicated in their construction and form. Some of these recreations may be the topics of study in a mathematical club, although even then they are devoid of the spontaneity which is the first prerequisite of the evoking of genuine interest.

What is then the correct approach to a mathematical recreation? How can a teacher resort to an introduction of a recreation with a modicum of certainty of success? Is there some definite process and procedure which will insure the teacher against the recreation falling on deaf ears? The answer to these questions requires careful consideration of the psychological effect of a mathematical recreation.

A mathematical recreation presupposes

a *lightness* of approach to a situation as well as a *lightness of the situation itself*. In other words, a mathematical recreation must be as light as a witty epigram. In formulating a mathematical recreation the teacher must consider several very important factors. Fundamentally the purely mathematical factors are of secondary importance when a classroom situation is considered. On the other hand, however, the pupils, their interest, their ages, their command of language, their responsiveness to humorous situations, all these must be carefully weighed and accounted for. In other words, whenever a mathematical recreation is about to be introduced in a mathematics classroom, the teacher should take care to avoid its presentation in purely mathematical form.

Generally speaking, the pupils are, by and large, very sensitive to humorous situations. Laughter is more conducive to effective learning than morbid and rigid seriousness, provided levity is avoided. The psychology of laughter is unfortunately taboo in educational theory. The readers of this department are advised to consult a little book by Henri Bergson on the topic of the psychology of laughter. The reason many of the books on mathematical recreations are usually food for the generals is very simple; those who wrote these books have taken themselves and their subject too seriously in a purely mathematical sense. The sense of humor of a bewhiskered mathematician is not of the same kind as the sense of humor of an average mortal pupil. And the table of mortality or of life expectancy of a pupil in a mathematical classroom is very sad indeed.

The above should not be interpreted as an exhortation to some new approach to mathematical instruction. Nor should it be construed as an appeal to the teachers to become jesters. There is no justification of any extreme. But, if a mathematical recreation is to be effective, it must appeal to the sense of humor of the pupil. And mathematics is not devoid of humor, if a teacher can develop an appropriate satirical technique. This department proposes to indicate some means which the teacher will be able to utilize in classroom situations. This can be accomplished without sacrificing the "greater values" of mathematical instruction, that is, the sacred cow which is known under the collective name of *objectives*.

Mathematics is not a difficult subject. It is, as a matter of fact, no more difficult than the everyday thinking we all are engaged in. The seeming and imaginary difficulty of mathematics may be ascribed to the outmoded traditional approach to the subject. If mathematical instruction is abundantly sprinkled with truly recreational material most of the fears of and for mathematics would vanish as the morning mist disappears under the rays of the rising sun. Recreational material is abundantly available and it beckons both the teacher and the pupil. Much of the recreational material, a greater portion of it, is found not in purely mathematical situations. But, whatever the source of this material may be, it must be clothed in a smile and a timely humorous dress.

Mathematical Miscellanea

(*Continued from page 398*)

and others for computing the area of any polygon from the coordinates of its vertices is also easily derivable. If (x_i, y_i) are the vertices of an n-gon, then its area may be computed from the array

$$A=\frac{1}{2}\begin{bmatrix} x_1 & & x_2 & & x_3 & \cdots & x_n & & x_1 \\ & \times & & \times & & & & \times & \\ y_1 & & y_2 & & y_3 & \cdots & y_n & & y_1 \end{bmatrix}$$

by summing all products of the form x_iy_{i+1} plus x_ny_1 and from this subtracting the sum of all products of the form y_ix_{i+1} plus y_nx_1 and finally taking one half of this result.

APPLICATIONS

Edited by Sheldon S. Myers
Department of Education, Ohio State University, Columbus, Ohio

We begin this first issue of 1951–52 with a renewed appeal for contributions. Those that you readers have submitted in the past have brought many favorable comments and have enabled The Mathematics Teacher, through its departments, to enrich the quality of mathematics instruction in this country. Perhaps this knowledge will encourage more of you to shift from passive to active beneficiaries of this journal.

P. G. 4. Gr. 10–12 *Runway Drop at Idlewild International Airport*

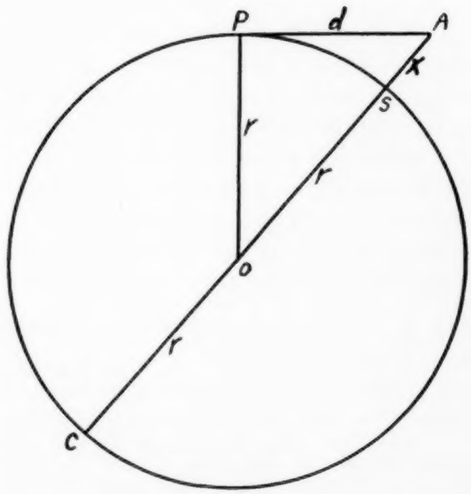
Dr. Maurice Nadler, Head of the Mathematics Department at Jamaica High School, Jamaica, New York, wrote me last April of an interesting application of the theorem in plane geometry which states that a tangent to a circle is perpendicular to the radius at the point of tangency.

In laying out a mile-long runway at Idlewild International Airport, there is a drop of 8″ due to the curvature of the earth. What would this drop be for a two-mile runway?

AP is the apparent level, tangent to the earth, while arc PS is the runway following the curvature of the earth. Since AP is tangent to OP, triangle AOP is a right triangle. Using the Pythagorean Theorem, it can be shown that

$$d^2 = (2r + x)x.$$

(Department Editor's Note: After the manuscript was completed, it was discovered that this was not the derivation intended by Dr. Nadler. He wished to point out the use of the following theorem which enables one to derive the above expression in one step: "If from a point without a circle a secant and a tangent are drawn, the secant terminating in the concave arc, the square of the tangent is equal to the product of the secant and its external segment.")



At this point a step occurs with which we would do well to acquaint our students. This is simplification when a very small, insignificant quantity is added to a huge quantity. In this case, x is negligible in comparison with $2r$, thus we have the following simplified expression:

$$d^2 = 2rx \quad \text{and} \quad x = d^2/2r.$$

Using the value of 7,900 for r, it is found that

$$x = 2/3\, d^2,$$

where d is in miles and x in feet.

For one mile, the drop, x, is $\frac{2}{3}$ of a foot or 8″. For two miles, the drop is 32″.

Dr. Nadler adds that a similar application is the development of the approximate formula for visibility from a given height above the earth's surface:

$$d = 1.3\sqrt{h},$$

where d is in miles and h in feet.

P. 3 Gr. 10–12 *A Valuable Dog is Lost by a Definition*

Many geometry teachers are interested in how transfer of training and motivation is achieved in Nature of Proof, a progressive method of teaching plane geometry developed by Dr. Harold P. Fawcett and described in the Thirteenth Yearbook of the National Council of Teachers of Mathematics. One of the techniques is to show dramatically the importance and the role of the definition in geometric and non-geometric situations. Teachers should be on the alert for newspaper clippings which will serve this purpose. Following is an Associated Press dispatch of June 6 from Sesser, Illinois, which might be used:

> Virgill Robbins decided to have his valuable pointer dog vaccinated against rabies last week but didn't have time to leave his hardware store. He asked a small boy to have it done and gave him a dollar for the fee.
>
> The lad took the dog to the City Hall and told Police Chief Wilford Day that Mr. Robbins wanted the dog "shot." "Surely not," Chief Day said, but when the boy pulled out the dollar for the job the chief guessed Robbins meant it. The police chief said he'd shoot the dog but would take no money.
>
> The boy returned to tell Mr. Robbins what he had done. The merchant rushed to the City Hall and found that his dog, for which he had spurned a $300 offer, had been destroyed.

A teacher could use this clipping to point up the following ideas in Nature of Proof:

1. The influence, on the conclusion, of a definition in an argument.
2. Definitions are human agreements, not eternal truths.
3. The need in mathematical and non-mathematical situations of group agreements about definitions and other assumptions in order to establish a common ground for communication and understanding.
4. The logical difficulties that might arise from implicit assumptions.

T. 2 Gr. 9–12 *Interpolation in the Laboratory*

One of the skills which is more often taught in trigonometry than in other areas and which has wide use in industry and science is interpolation. It is important that students do not conclude from too limited experience with interpolation that its use is confined to logarithms and trigonometric functions. Other uses can easily be shown to high school students. Here is one involving the immersion refractometer which determines the refractive index of liquids like oil in order to get a line on their density.

The immersion refractometer is immersed in the oil, and the bending of a a ray of light by the oil is measured by the instrument. The amount of bending, or refractive index, is a measure of the density of the oil. However, the refractometer readings are in terms of scale divisions and must be converted to refractive index. This conversion may be done with the aid of a table in Lange's *Handbook of Chemistry*.[1] This table gives refractive indexes ranging from 1.32539 to 1.36640 for instrument readings ranging by whole numbers from −5 to 105. Here is a sample interpolation for an instrument reading of 20.8:

Scale Divisions	Refractive Index
20.0	1.33513
20.8	?
21.0	1.33551

For the purpose of interpolating, it is correct to assume, in this interval, that refractive index is linearly related to instrument reading. Therefore, since 20.8 is .8 of the way between 20.0 and 21.0, the refractive index corresponding to 20.8 must be .8 of the way between the first and second refractive index, or .8 of .00038. This is .00030 units of refractive index beyond the first one, or 1.33543. As in all mathematics teaching, students should be taught how to generalize processes and how to use them in different situations. Experimental studies have shown that this is the best way to assure transfer of training.

(*Continued on page 404*)

[1] Norbert Adolph Lange, *Handbook of Chemistry*. (Sandusky, Ohio: Handbook Publishers Inc., 1941. p. 1087.)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, N. Y.

Roman Numerals

THE TOPIC of Roman numerals appears in textbooks in arithmetic for its informational value. It is handy to have a second set of numerals, and tradition links us to the Roman numerals for this auxiliary set. Custom dictates their use on the face of a clock. Convenience causes their use after the identification number on a United States Savings Bond to indicate the maturity value,—L on the $50.00 bonds, C on the $100.00 and D on the $500.00 (The $25.00 bond is marked Q.)

Pupils studying these numerals have a tendency to experiment with them, sometimes departing widely from the accepted pattern. Indeed instances may be found of out-of-school eclectic uses of the numerals. In 1949, for example, an Italian postage stamp was issued in honor of the physicist Volta. It bore the dates MDCCIC and MCMIL.

Should the innovations made by the pupils be accepted? and how may we answer the question "Is there a way to write really *big* numbers?" Prudence dictates that for testing purposes, the pupil should stay with the textbook and its rules. Common sense says that there must have been ways to write large numbers for the Romans would have needed them in their censuses and in other statistics. Reference to Professor Florian Cajori's *History of Mathematical Notations*, vol. I (Chicago, 1928) and to Professor D. E. Smith's *History of Mathematics*, vol. II, (Boston, 1925) shows a wide variety of forms used by the Romans. Listing here, however, is no indication of the vogue of a symbol nor is the date of its occurrence an indication of the time when it was first introduced. Professor Smith notes that the Romans appear to have avoided the symbol IV for 4 preferring IIII instead. Professor Cajori states that a line above a numeral to indicate multiplication by 1000 appears in an example of the 4th century A.D. Among the special instances listed is the numeral XXCIII instead of LXXXIII.

Roman numerals were in active use for a considerable period in Europe. They were used in an inventory of stores in the English navy in the second quarter of the seventeenth century. It is said that they were used in government accounts in France until 1789 at the official beginning of the French Revolution. This, however, should be checked for it would seem incredible. It perhaps was the fact that the numerals were in frequent use as matters of record that led certain of the writers of arithmetics to omit mention of them. In this group are Tonstall and Humphrey Baker in the sixteenth century and Hodder and Leybourne in the seventeenth. In the arithmetics printed in the United States, beginning with Pike's of 1788, Roman numerals are generally given at least half a page although the topic is dropped at this point. One of the most complete treatments is given by Roswell C. Smith in his *Arithmetic on the Productive System* (Rochester, 1846). His treatment of the subject lists the numerals without giving rules for their formation except in the case of two short footnotes.

The topic is headed

NUMERATION

"NUMBER, which shows how many are

meant, is represented by letters, by words, and by characters called figures as:—

One	I	1
Two	II	2
Three	III	3."

The author gives each number through 16 then takes multiples of 10 to 100, multiples of 100 to 1000, multiples of 1000 to 10,000, then 20,000, 50,000, 100,000, and 1,000,000.

A footnote accompanies IV, IX, XL, XC, and DCCCC giving the alternate symbols "Or, IIII for 4; VIIII for 9; XXXX for 40, LXXXX for 90 and CM for 900." The number 400 is given as CCCC with no alternate. The number 500 introduces a second symbol IↃ which is used consistently thereafter. The symbol for 1000 may be M or CIↃ. Two thousand may be MM or $\overline{\text{II}}$. Seven thousand is $\overline{\text{VII}}$ or IↃↃ MM. The second footnote states that "Every Ↄ annexed to IↃ increases its value 10 *times*: as IↃ is 500, IↃↃ is 5000: in like manner, the prefixing of C and the annexing of Ↄ to CIↃ increases it 10 *times*, as CIↃ 1000, CCIↃↃ 10,000; lastly, a line over any number increases it 1000 *times:* as D, 500, $\overline{\text{D}}$, 500,000." Following the footnote, the number 1,000,000 appears as $\overline{\text{M}}$ or CCCCIↃↃↃↃ.

The use of CCCC for CD occurred in a number of the books that were consulted and it may be inferred that this was fairly common.

The treatment of Roman numerals by Roswell C. Smith shows an extension that twentieth century texts definitely lack,—the notation for large numbers. While this has little use if any, it serves at least to answer the question "How do you indicate a *big* number in Roman numerals?" As for the other problem "Why can't I write 8 as IIX or 49 as IL?" the unanswerable situation is that people have done so at different times, but it is better (textbook) form to write VIII and XLIX.

Applications

(*Continued from page 402*)

Ar. 10 Gr. 7–10 Estimating the Age of the Earth

Here is a timely and prophetic guide-sheet used in 1943 in the University School, containing the initials of Dr. John Kinsella.

1. Scientists have discovered that *uranium* slowly changes into *lead*. One ounce of uranium will, in millions and millions of years, finally change into 0.8653 ounce of lead, and 0.1345 ounce of helium, while 0.0002 ounce is converted to radiant energy.
2. Scientists have estimated the following time-table for the radioactive disintegration of one ounce of uranium:
   a. after 100 millions years—0.985 oz. U—0.013 oz. lead
   b. after 1000 millions years—0.865 oz. U—0.116 oz. lead
   c. after 2000 millions years—0.747 oz. U—0.219 oz. lead
   d. after 3000 millions years—0.646 oz. U—0.306 oz. lead.

   The amounts of uranium and lead in an ounce of the mineral, *uraninite*, can be determined very accurately by quantitative chemical analysis. These amounts will indicate how long the process of disintegration has been going on. This process is steady and continuous, unaffected by changes in temperature, pressure, and chemical changes. Thus, with a more detailed time-table and chemical analysis, we have a perfect clock for measuring the age of the mineral.
3. By this and other less exact methods, the earth has been estimated to be at least 1,500,000,000 years old. The name of this number is __________ years.
4. Following are some estimates of the earth's age:

Estimator	Estimate (years)
a. Kovarik, Yale University	1,852,000,000
b. Lord Rutherford, England	3,400,000,000
c. Jeans, England	2,000,000,000.

   Write the name of the average of the above estimates. __________ years.
5. The oldest living things on earth today are the gigantic Sequoia redwood trees in California. The General Sherman Tree in Sequoia National Forest is estimated to be between 3,000 and 4,000 years old. Let us assume the age of this tree to be 3,500 years. Look up some historical events which were taking place this long ago. Assume the earth to be 2,415,000,000 years old and the oldest living thing to be 3,500 years old. Then the earth is how many times as old as the oldest living tree? __________. The name of this number is __________.

DETERMINANTS-A POWERFUL TOOL

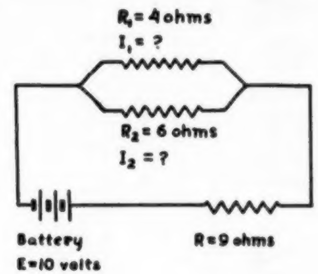
THE SOLUTION OF SIMULTANEOUS EQUATIONS IN ELECTRICAL CIRCUITS

THE MATH:

$$13x + 9y = 10$$
$$4x - 6y = 0$$

$$x = \frac{\begin{vmatrix} 10 & 9 \\ 0 & -6 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = \frac{-60}{-114} = .526$$

$$y = \frac{\begin{vmatrix} 13 & 10 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = \frac{-40}{-114} = .351$$



R_1 = Resistance in top branch
I_1 = Current " " "
R_2 = Resistance in bottom branch
I_2 = Current " " "
R = Resistance in main line

ITS APPLICATION TO THE ELECTRIC CIRCUIT:

FROM CIRCUIT THEORY

$$I_1 R_1 + (I_1 + I_2) R = E$$
$$I_1 R_1 - I_2 R_2 = 0$$

SUBSTITUTING THE VALUES IN THE DIAGRAM

$$13 I_1 + 9 I_2 = 10$$
$$4 I_1 - 6 I_2 = 0$$

$$I_1 = \frac{\begin{vmatrix} 10 & 9 \\ 0 & -6 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = .526 \text{ amperes}$$

$$I_2 = \frac{\begin{vmatrix} 13 & 10 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 13 & 9 \\ 4 & -6 \end{vmatrix}} = .351 \text{ amperes}$$

PREPARED IN COLLABORATION WITH THE MENS AND WOMENS' MATHEMATICS CLUBS OF CHICAGO

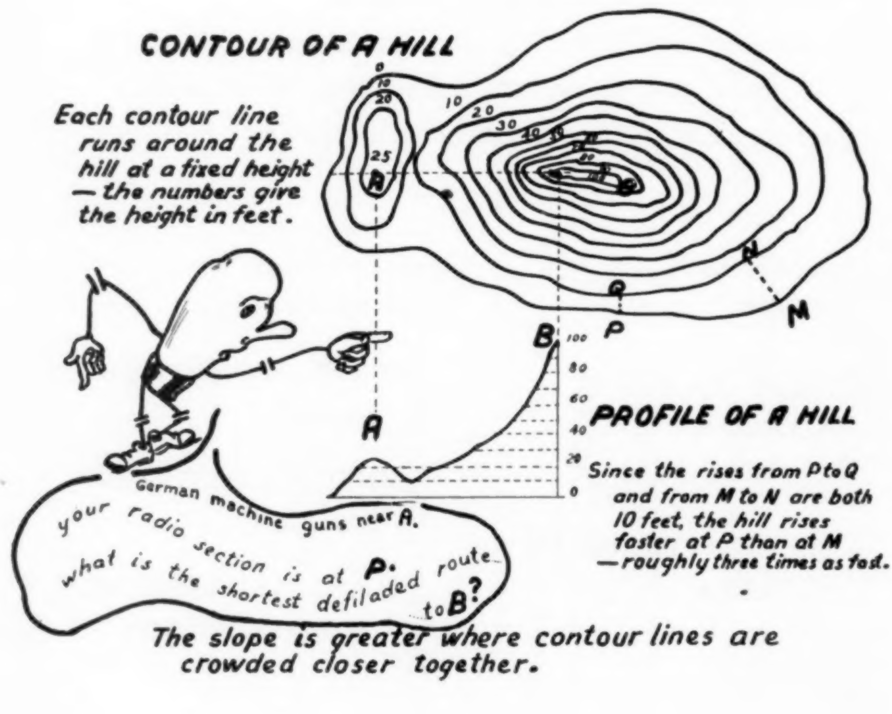
PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER # 8

SLOPES IN MAP READING

SIGNAL MEN READ MAPS

A knowledge of slopes is useful in obtaining the outline of a hill from its contour lines in the map.



The slope is greater where contour lines are crowded closer together.

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POSTER # 7

THE MATHEMATICS OF RADIO

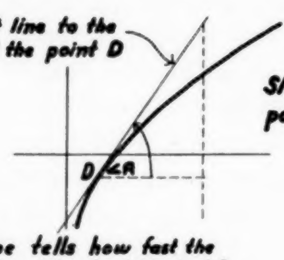
SLOPES

An example of the slope of a curve is the grade of a road.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \tan \angle A$$
(or grade)

The slope tells how steeply the road is rising or falling.

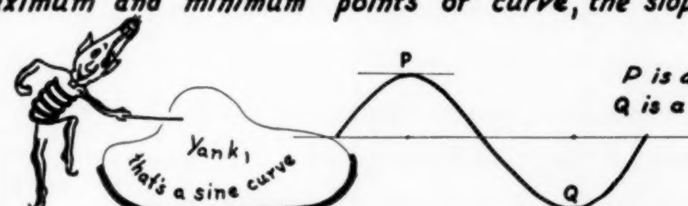
For a general curve we use tangent lines to find the slope.



Slope of curve at point D = tan ∠A

An important use of slopes is to find maximum points (peaks) and minimum points (valleys) of a curve.

At maximum and minimum points of curve, the slope is zero.



P is a maximum
Q is a minimum

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PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER #6

the
MATHEMATICS
of *RADIO*

GRAPHS

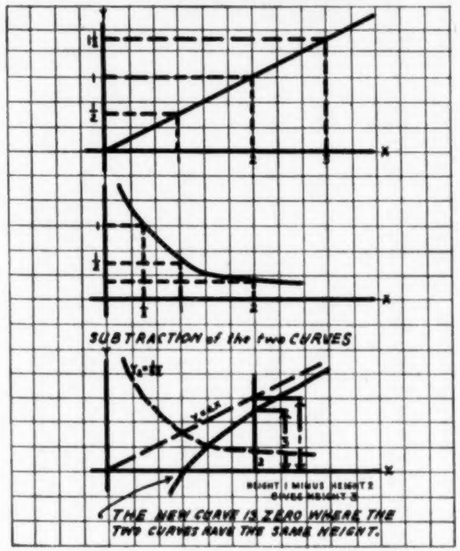
the EQUATION

$$Y_1 = a X$$

$$Y_2 = \frac{1}{bX}$$

$$Y_1 - Y_2 = aX - \frac{1}{bX}$$

the GRAPH



PLOTTING the CURVE

Let $a = \frac{1}{2}$ for this case

IF X IS	THEN Y IS
0	0
1	$\frac{1}{2}$
2	1
3	$1\frac{1}{2}$

Let b=2 for this case

IF X IS	THEN Y IS
0	∞ (Infinity)
$\frac{1}{2}$	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

For each X, we get the height of the new curve by subtracting the height of curve 2 from the height of curve 1.

IT'S THE SIGNAL CORPS FOR ME

PREPARED IN COLLABORATION WITH THE MEN'S AND THE WOMEN'S MATHEMATICS CLUBS OF CHICAGO

PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER #8

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John R. Mayor
The University of Wisconsin, Madison, Wisconsin

In this issue this Department is given over entirely to data recently made available by the Federal Security Agency, Office of Education. The tables given below are taken from the Statistical Circular, No. 294, National Summary of Offerings and Enrollments in High School Subjects, 1948–49, by Mabel C. Rice, Survey Statistician. The study was carried out under the joint auspices of the Research and Statistical Service and the Division of Elementary and Secondary Education. This section includes those parts of Tables 1 and 2 which refer to enrollments in mathematics, except for the 7th and 8th grades, and a limited selection from other parts of these Tables so that enrollments in mathematics may be compared with those in other areas. The enrollments in 7th and 8th grade mathematics are

Table 1.—*Number and Percentage of Pupils Enrolled in High-School Subjects, and Number of States* Reporting Each Subject, 1948–49*

(All public secondary day schools, continental United States.)

Subject-field and subject	Number of States* reporting subject	Enrollment	
		Number	Percent[1]
1	2	3	4
Mathematics			
Elementary algebra	49	1,042,451	15.1
Intermediate algebra	49	372,152	5.4
General mathematics	49	649,810	9.4
Plane geometry	49	599,336	8.7
Solid geometry	49	93,944	1.4
Trigonometry	49	108,551	1.6
Advanced or college algebra	46	34,363	.5
Advanced general mathematics	44	42,600	.6
Mathematics review	24	12,332	.2
Analytics	9	468	[2]
Calculus	3	185	[2]
Other mathematics	20	1,617	[2]
English			
Ninth grade English	49	1,564,358	22.6
Tenth grade English	49	1,397,897	20.2
Eleventh grade English	49	1,198,018	17.3
Twelfth grade English	49	855,617	12.4
Speech and public speaking	49	246,213	3.6
Social Studies			
U.S. history, advanced	49	1,231,694	17.8
World history (grades 9–12)	49	876,432	12.7
World geography (grades 9–12)	49	271,969	3.9
American government or advanced civics	48	431,916	6.3
Problems of democracy	48	282,971	4.1
Economics	49	254,770	3.7
Science			
Ninth grade—general science	49	1,073,934	15.5
Biology	49	989,756	14.3
Chemistry	49	406,662	5.9
Physics	49	278,834	4.0

* In this study, the District of Columbia was counted as a State.
[1] Based on the total enrollment of 6,907,833 pupils in all public secondary day schools, 1948–49.
[2] Less than .05 percent, or less than 1 pupil in 2,000.

omitted since these grades are included in this study only when they are organized as a part of the secondary school.

Subjects other than mathematics, which show noticeable increases and decreases for the sixteen year period, have been selected from Table 2. The study reveals decreases on a percentage basis in enrollments in ancient and medieval history, physics, Latin, French, bookkeeping, shorthand, and economic geography, and increases over the same period for U. S. History, world history, geography, problems of democracy, general science, biology, Spanish, typewriting, home economics, agriculture, industrial subjects, physical education and music.

The following paragraphs, descriptive of coverage and procedures in the study, are quoted directly from The Statistical Circular, No. 294.

> This circular presents national information on offerings and enrollments in high-school subjects for all public secondary day schools for the school year 1948–49 (Table 1). In addition,

(*Continued on page 421*)

TABLE 2.—*Number and Percentage of Pupils Enrolled in Subjects Taught in the Last 4 Years of High School in 15 or More States,*[1] *1948–49, and Percentage Enrolled in 1933–34*

(Public Secondary Day Schools, Continental United States)

Subject	1948–49		1933–34 Percent[3]
	Number	Percent[2]	
1	2	3	4
Mathematics			
Algebra	1,448,966	26.8	30.4
General mathematics	704,742	13.1	7.4
Plane geometry	599,336	11.1	15.1
Solid Geometry	93,944	1.7	2.0
Trigonometry	108,551	2.0	1.3
Other Subjects			
U.S. history, advanced	1,231,694	22.8	17.3[4]
World history	876,432	16.2	11.9
Ancient and/or medieval history	79,473	1.5	11.0
Geography	301,652	5.6	2.1
Problems of democracy	282,971	5.2	3.5
General science	1,121,980	20.8	17.8
Biology	995,930	18.4	14.6
Physics	291,473	5.4	6.3
Spanish	443,995	8.2	6.2
Latin	422,304	7.8	16.0
French	255,375	4.7	10.9
Home Economics	1,304,846	24.2	16.7
Agriculture	364,185	6.7	3.6
Industrial subjects	1,434,302	26.6	21.0
Physical education	3,747,220	69.4	50.7
Music	1,625,235	30.1	25.5
Bookkeeping	472,163	8.7	9.9
Typewriting	1,216,142	22.5	16.7
Shorthand	421,635	7.8	9.0
Economic geography	90,045	1.7	4.0

[1] In some instances the elementary, applied or advanced courses included with a basic subject (see note) were not taught in as many as 15 States.

[2] Based on total enrollment of 5,399,452 in the last 4 years in public secondary day schools in the United States.

[3] Based on total enrollment of 4,496,514 in the last 4 years in 17,632 public secondary day schools in the United States reported in the 1933–34 survey.

[4] Includes enrollments in grades 9–12. Data for 1948–49 are for grades 10–12 only.

NOTE.—For improved historical comparability, the enrollments in all levels of a subject (elementary, applied, or advanced) have been combined with the enrollment in the basic subject. Thus, the enrollment in "algebra" includes the enrollments in elementary algebra, intermediate algebra, and advanced or college algebra.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

An Angle Bisector Device[1]

The angle bisector device suggested here involves the same principles for bisecting an angle as are employed in the usual ruler and compasses construction. The device itself is of interest because it helps add a note of generality to the usual geometrical construction, and also because it offers possibilities for several nice applications.

The device consists of four bars which may be cut from cardboard, wood, metal, or plastic. (See Fig. 1.) If cardboard is used the device can be assembled with brass paper fasteners. If the heavier materials are used $\frac{1}{8}''$ brass bolts and washers should be used. Suggested dimensions for the device are as follows: OR and OT, each 20″ long; AF and BF, each 9″; and OS, 24″. As is illustrated by the diagram, bar OS must have a slot cut in it. In order that the device may be used to bisect a large range of angles, the slot in OS should be made as long as the strength of the material will permit.

To assemble the device, first locate A and B on bars OR and OT respectively so that $OA = OB = 7\frac{1}{2}''$. Drill holes at O, A, B, and F on the various bars as needed, insert bolts and washers as needed, and

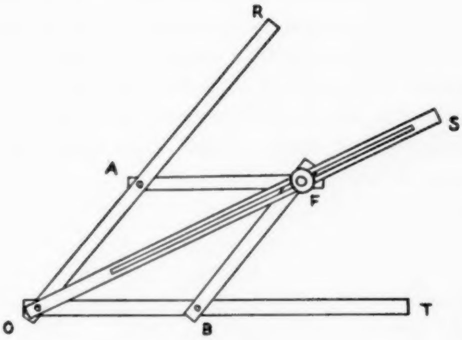


Fig. 1

the construction is complete. Since triangles BOF and AOF are congruent for all positions of F along OS, the device can be used to bisect any angle AOB within the limits allowed by the mechanical features of the instrument.

An examination of the device will reveal that it can also be used to demonstrate the principles involved in constructing the perpendicular bisector of a line segment. If A and B are placed at the extremities of a given line segment, then OF will be its perpendicular bisector. As another application the device may be used to find the center of a circular object. Suppose that we wish to find the center of the circular end of a log. If arms OR and OT are closed until both are tangent to the circle, then OS will pass through its center. By making a pencil mark along the slot, rotating the position of the device around the log, and then making a second mark again along the slot, the center of the end of the log is easily located. The proof of this fact depends upon the truth of the theorem which states that the bisector of the angle formed by two tangents to a circle from a point outside the circle passes through the center of the circle.

[1] The three devices described in this issue were designed and produced by tenth grade plane geometry students in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota.

Incenter Demonstrator

By applying the mechanical principles of the angle bisector device to the problem of bisecting the three angles of a general triangle simultaneously, it is possible to develop a second device which will show that the three angle bisectors are concurrent. This is not difficult, nor is the new device as complicated as it might appear from a casual inspection of the diagram. (See Fig. 2.) The device consists essentially of three angle bisectors, one mounted at each of the vertices of the general triangle ABC.

Fig. 2 the following dimensions are recommended: *DF*, *EF*, *HK*, *MK*, *RS*, and *ST*, each 4″ long; *AG* and *BN*, each 10″; *PC*, 14″; *AB*, 12″; *AC*, 17″; *BX*, 22″; and *TC*, 3″. The dimensions listed are generous enough so that holes needed to assemble the device may be drilled $\frac{1}{4}$″ from the ends of the bars. To facilitate reference to the diagram during construction all bars should be lettered to correspond with the diagram as soon as they are cut to size. All holes drilled in the various bars should be large enough to accommodate a $\frac{1}{8}$″ brass bolt.

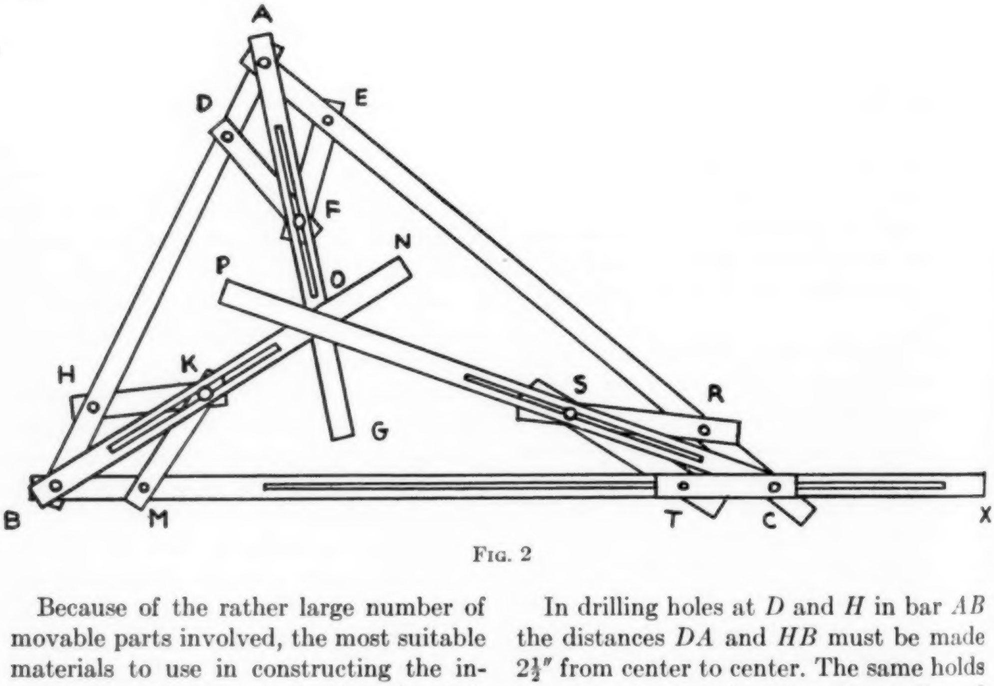


Fig. 2

Because of the rather large number of movable parts involved, the most suitable materials to use in constructing the incenter demonstrator are plastic and metal; however, it is also possible to use wood. An enterprising student in the department editor's plane geometry class last year built the device with laths which he obtained from a dismantled wooden venetian blind. The movable parts worked beautifully.

Unfortunately it is not very practical to use the dimensions suggested in the previous article to build the incenter device, because it would become too large to manipulate. For the device illustrated in

In drilling holes at D and H in bar AB the distances DA and HB must be made $2\frac{1}{2}$″ from center to center. The same holds true for the lengths MB, CR, TC, and AE. Slots cut in bars AG, BN, and CP need not be made more than 4″ long, nor need they be cut closer to A, B, and C than $2\frac{1}{2}$″.

The device can be assembled in much the same manner as the angle bisector device. There is only one difference; bar TC is a sliding bar. Both its ends slide along the slot in bar BX. This feature, however, presents no special difficulty. If all parts have been cut according to instructions, the single sliding bar TC will

fit into position automatically. To demonstrate that the angle bisectors are concurrent for any triangle which can be produced with the device simply slide the vertex C along the slot in BX.

A Harmonic Divider

Another device which may be built by applying the basic ideas of the angle bisector device is what might be called an harmonic divider. Its construction is based on a direct application of the theorem which states that the bisector of an interior angle of a triangle and the bisector of the adjacent exterior angle divide the opposite side harmonically. For triangle ABC this means that if AE and AF are respectively the bisectors of angles BAC and CAH, then $BQ/QC = BP/PC$. (See Fig. 3.)

The trick in the construction is to develop a system of movable parts such that AE and AF will bisect angle BAC and the adjacent exterior angle simultaneously, while at the same time leaving bar AD free to intersect bar BM anywhere along its length.

It's really quite simple—almost as simple as building the angle bisector device alone. An inspection of the diagram will reveal that the two bars AE and AF are all one piece. They are joined together in the same manner as the arms of a carpenter's square. If a hole is drilled in the "square" at A, then both arms will pivot together always 90° apart. But the bisectors of an interior and adjacent exterior angle of a triangle are perpendicular to each other, so if AE can be made to bisect angle BAC for all positions of C along BM, then AF will automatically bisect angle CAH. Angle BAC can be bisected mechanically for all positions of C by mounting an angle bisector device at A which uses bar AE of the "square" as its slotted bisector bar.

The "square" may be constructed of wood, sheet metal, or plastic. If wood is used the two bars AE and AF may be screwed to the "leg" edges of a small draftsman's triangle in order to form the necessary right angle, or they may be screwed to the arms of a small flat angle iron. If sheet metal or plastic are used, the "square" can be cut out as one piece in the same manner as the carpenter's square. Incidentally, sheet aluminum works excellently. Convenient dimensions for AE and AF are 10″ and 20″ respectively. Suggested dimensions for the other bars of the device are as follows: BH and AD, each 12″ long; OR and OS, each $3\frac{1}{2}$″; and BM, 24″.

(*Continued on page 421*)

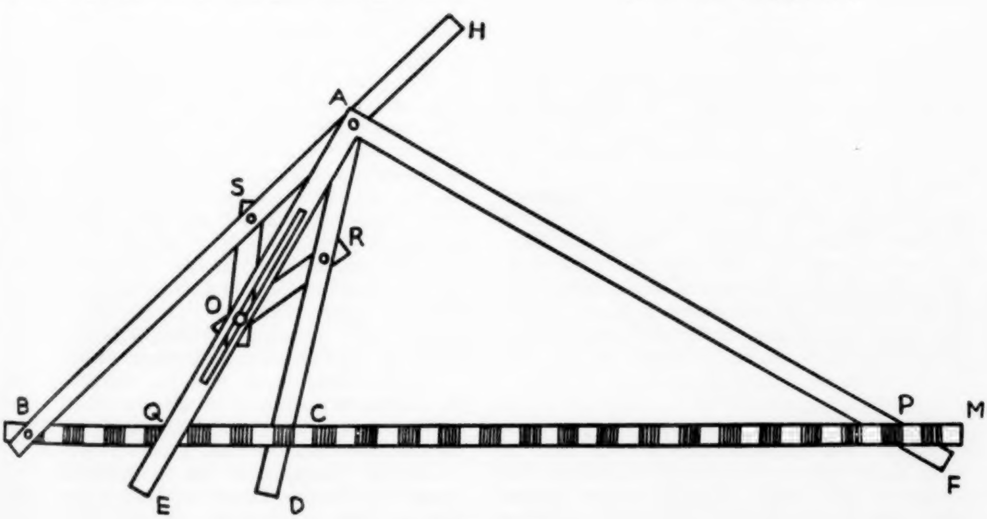


Fig. 3

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

ELEMENTARY SCHOOL

Row-Peterson Arithmetic, Primer, by Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. With Workbook. Paper, 64 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. Primer, $0.60; Workbook, $0.40.

Row-Peterson Arithmetic, Book One, by Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. With Workbook, Paper, 96 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. Book One, $0.72; Workbook, $0.40.

Manual, Row-Peterson Arithmetic, Primer and Book One, by Margaret Leckie Wheat and Harry G. Wheat. Paper, 144 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $0.60.

Row-Peterson Arithmetic, Book Two, by Harry G. Wheat, Geraldine Kauffman and Harl R. Douglass. With Workbook. Cloth, 240 pages, 1951. Row-Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $1.72; Workbook, $0.48.

Manual, Row-Peterson Arithmetic Book Two, by Margaret Leckie Wheat and Harry Grove Wheat. Paper, 188 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $0.60.

HIGH SCHOOL

1. Special Courses in Mathematics

Machine Shop Mathematics, Second Edition, by Aaron Axelrod, New York University. Cloth, xi+359 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $3.60.

Practical Mathematics, Part IV, Trigonometry and Logarithms by Claude I. Palmer, late Professor of Mathematics and Dean of Students, Armour Institute of Technology; and Samuel F. Bibb, Illinois Institute of Technology. Fifth Edition. Cloth, xii+193 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $2.60.

COLLEGE

1. Algebra

Algebra for Commerce and Liberal Arts, by Alvin K. Bettinger, and Wendell A. Dwyer, The Creighton University. Cloth, xi+225 pages, 1951. Pitman Publishing Corporation, 2 West 45th Street, New York, N. Y. $3.00.

Intermediate Algebra, by Paul K. Rees, Louisiana State University; and Fred W. Sparks, Texas Technological College. Cloth, viii+328 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $3.25.

College Algebra, by Henry L. Rietz, late of the University of Iowa; Arthur R. Crathorne, late of the University of Illinois; Fifth Edition revised by J. William Peters, University of Illinois. Cloth, xv+387 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York, 10, N. Y. $2.95.

2. Analytic Geometry

Analytic Geometry, Second Edition, by John W. Cell, North Carolina State College, Cloth, xii+326 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.75.

Brief Course in Analytics, Revised, by M. A. Hill, Jr., and J. B. Linker, University of North Carolina. Cloth, xi+224 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.40.

3. Statistics

Index Numbers, by Bruce D. Mudgett, University of Minnesota. Cloth, x+135 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.00.

4. Advanced Mathematics

Differential Equations, Third Edition, Revised, by H. B. Phillips, Massachusetts Institute of Technology. Cloth, viii+149 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.00.

Linear Computations, Paul S. Dwyer, University of Michigan. Cloth, xi+344 pages, 1951. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $6.00.

Introduction to the Theory of Algebraic Functions of One Variable, by C. Chevalley. Mathematical Surveys Number VI. Cloth, xi+188 pages, 1951. American Mathematical Society, 531 West 116th Street, New York. $4.00.

Tables Relating to Mathieu Functions, by the National Bureau of Standards. Cloth, xlvii+278 pages, 1951. Columbia University Press, 2960 Broadway, New York 27, N. Y. $8.00.

Fourier Transforms, Ian N. Sneddon, University of Glasgow. Cloth, xii+542 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $10.00.

Teaching of Mathematics

How to Teach Arithmetic, by Harry G. Wheat, West Virginia University. Cloth, vii+438 pages, 1951. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Illinois. $3.00.

Teaching the New Arithmetic, Second Edition, by Guy M. Wilson. Cloth, xiv+438 pages, 1951. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. $4.50.

REVIEWS

Cotton Mill Mathematics (Second Edition), Thomas H. Quigley and W. S. Smith. Atlanta, Turner E. Smith and Co., 1930. xiv+322 pp., $2.67.

This text is prepared for use in the classroom by school age as well as adult students who have an intimate knowledge of the cotton mill but who have little mathematical background and yet wish to further themselves in the industry.

The book is divided into two parts. Part I deals with basic arithmetic. It covers sequentially fundamental operations from addition of whole numbers through fractions, decimals and percentage. Throughout this part there are generous practical application problems concerning the various specializations of a cotton mill.

Part 2 has no set sequence but is divided into chapters to permit selection of more technical topics posed by each of the specializations in a cotton mill.

There is no index, but there is a table of contents listing the chapter titles. These chapters are numerous and each one covers a very limited phase of the text. Answers to all exercises are included.—John T. Ladd, Highland Park High School, Highland Park, Michigan.

Essentials of Business Arithmetic (Third Edition), Edward M. Kanzer and William L. Schaaf. Boston, D. C. Heath and Company 1950. viii+476 pp., $2.36.

This attractive and well-written text is planned for a full-year course of study but may be used conveniently for a shorter course. The first half of the book embodies the subject matter of elementary business arithmetic developed in one-lesson units which can be completed in approximately 85 class periods. These units are introduced by means of simple, concrete problems based on actual business activities; much care is taken to familiarize the student with business procedure and terminology in order to increase understanding and insight. The second half of the book contains important supplementary material.

A feature of the text is a wealth of good exercises. These are classified into A, B, and C assignments to provide for individual differences. Topical reviews and cumulative reviews are included at frequent intervals. General review problems and specimen examinations are given at the end of the book.

The reviewer recommends this text to the attention of all teachers of business arithmetic. —Harold D. Larsen, Albion College, Albion, Michigan.

Plane and Spherical Trigonometry (Fifth Edition), Claude I. Palmer, Charles W. Leigh and Spofford H. Kimball. New York, McGraw-Hill Book Company, Inc., 1950. xiii+368 pp., $3.25.

The order of topics has been maintained, in this fifth edition, including the early introduction of radian measure, inverse functions and equations. This is a commendable feature, as many teachers follow the text and if these topics are included in the latter part of the book they are omitted from the student's experiences.

Instead of reducing angle to the explicit form $n\cdot 90°+\theta$ in finding values of functions of positive angles larger than 90°, or negative angles, use is made of the acute angle between the terminal side of the angle and the x-axis. The treatment of the tan 90° is very well handled and should avoid the situation in which the student believes tan 90° exists and is equal to the number infinity, which is not the case.

The text is well organized, diagrams are plentiful and well labeled, important concepts are given in bold type, formulas are derived for the students, there are many practical applications and related problems included throughout the text with an entire chapter devoted to this phase of the work and greater emphasis is given to the curves of the functions and less to line representation.

The discussion of logarithms is the final chapter in the text. This will necessitate a change in the sequence of teaching as logarithms are used in solving triangles in previous chapters. An alert and skillful teacher will have no difficulty with this situation. The book does not stress logarithms, which is so common in many trigonometry texts.

The book could be used by a skillful teacher in high school, but I believe it is better suited for more mature minds.—George L. Keppers, Iowa State Teachers College, Cedar Falls, Iowa.

Plane and Spherical Trigonometry (Revised Edition), John A. Northcott. New York, Rinehart and Company, Inc., 1950. ix+234 pp., $3.50.

Dealing with a book that is in its revised edition, we know that changes that were made have been suggested by experience. This text certainly shows the experience of many teachers who have suggested that more be done to help students realize the great importance of the analysis of trigonometric functions.

Here the plotting of trigonometric curves as well as the inverse functions are given their rightful importance. An abundance of figures and well-worded and clear explanations further enhance the worth of this volume for those stu-

dents who plan to continue their mathematical studies in analytic geometry and the calculus.

Problems in the solution of triangles, although given later in this text than in many others, are plentiful and will give sufficient practice to those who plan to use a knowledge of trigonometry almost immediately after completing the course.

The section dealing with spherical trigonometry has been left almost the same as in the first edition. Excellent treatment of the topics chosen in this section is what undoubtedly suggested that this section remain as it was in the beginning.

Students who use this text will be helped greatly in reaching the important first steps of what is so often called "Mathematical maturity."—REV. BONAVENTURE KNAEBEL, St. Meinrad Minor Seminary, St. Meinrad, Indiana.

Plane and Spherical Trigonometry, H. L. Rietz, J. F. Reilly, and Roscoe Woods. New York, The Macmillan Company, 1950. xiii+204 pp., $3.00. Without tables, $2.75.

The authors of this text recommend its use in technical schools and first year college programs. It would seem possible that high school teachers could use this book to advantage in any class having capable students. No attempt is made to create interest in trigonometry through appeals to the student; instead, significant ideas are presented in problems designed to arouse the native interests of the mathematically inclined. College instructors will find simplified presentations to be within the grasp of the majority, whereas this is not necessarily true for high school-age groups. The teacher will need to supply items of historical and contemporary nature having interest value. Items of explanation do not attempt applications, but are devoted to brief, concise, discussions of various topics. Many exercises pertain to physics and problems workmen might encounter. Selectivity is possible because each set of exercises contains more problems than are necessary. Answers to the odd numbered problems are included.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Topology of Manifolds, R. L. Wilder. New York, American Mathematical Society, 1949. ix+402 pp. $7.00.

This book is volume 32 in the Colloquium Publications of the American Mathematical Society. As is customary for books of this series, the reader is expected to have reached a rather advanced stage of mathematical maturity. However, it is true that the book is essentially self-contained in that no technical knowledge is assumed as a prerequisite.

There are twelve chapters of which at least half are introductory in the sense that they develop the concepts and techniques necessary for stating and solving the problems of interest to the author. The first chapter constitutes an introduction to topology and also contains historical remarks on topology. Chapter V introduces the algebraic topology needed for later chapters including a discussion of homology theory, which is basic for what follows. The results in the final chapter are almost entirely due to the author and most of them are new. An appendix on unsolved problems and detailed bibliographical comments are included.—D. H. POTTS, Northwestern University, Evanston, Illinois.

The Location of Critical Points of Analytic and Harmonic Functions, J. L. Walsh. New York, American Mathematical Society, 1950. viii +384 pp., $6.00.

This book is volume 34 of the Colloquium Publications series. The author uses the term critical point to include both zeros of the derivative of an analytic function of a complex variable and points where the two first partial derivatives of a harmonic function vanish. The problem investigated is: given a function, determine the regions in the complex plane which contains all (or none) of the critical points. The subject has its historical origin in the work of Gauss. A typical example of the results is the theorem of Lucas: Let $p(z)$ be a polynomial and let R be the smallest convex set on which the zeros of $p(z)$ lie. Then all the zeros of the derivative $p'(z)$ also lie on R.

The avowed purpose of the author was to assemble and unify much of the known material on the subject, but results which are new have also been included. Although many of the problems are of a topological nature the methods employed are those of classical analysis. Accordingly the requisite background is a course on functions of a complex variable.—D. H. POTTS, Northwestern University, Evanston, Illinois.

Mathematics of Relativity, G. Y. Rainich. New York, John Wiley and Sons, Inc., 1950. vii +173 pp., $3.50.

This book is an extremely fine introduction to the mathematics of relativity. All mathematical tools beyond elementary differential equations are developed within the book. However, a certain amount of background in college physics is essential. An outline of the presentation can roughly be given as follows. In Chapter 1 certain basic mathematical expressions of classical physics are put into a new form. This new form suggests a four-dimensional geometry, which is then developed in Chapter 2. Chapter 3 is entitled "Special Relativity" and explores the consequences of the application of four-dimensional geometry to physics. Defects of this application are pointed out and the generalization to curved spaces is made in Chapter 4 to rectify the situation. The application of the geometry thus obtained to physics is carried out in Chapter 5 on "General Relativity."

As the author points out, most of the material is not new. But there is one important exception. It is shown, contrary to previous beliefs, that the original Einstein theory is ade-

quate for both gravitational and electromagnetic phenomena. The question of quantum phenomena is still left open.

Although the reviewer is in agreement with the author on the point that "complete clarity in presenting the theory of relativity can be achieved only by stressing the mathematical aspect," it was felt that the physical and philosophical aspects could have stood some amplification. Certainly a knowledge of the historical background of the theory of relativity would increase the reader's grasp of the situation.—D. H. POTTS, Northwestern University, Evanston, Illinois.

Theory of Sets. E. Kamke. Translated from the second edition of *Mengenlehre* by F. Bagemihl. New York, Dover Publication, 1950. viii+152 pp. $2.45.

This is an excellent translation of a very fine book. It is a readable introduction to the general theory of sets, and is liberally supplied with examples. Even an almost total lack of background would not prevent an interested student from learning a great deal about the subject.

After discussion of operations with sets, and of the concepts of enumerability and non-enumerability, the author takes up in succession arbitrary sets and their cardinal numbers, ordered sets and their order types, well-ordered sets and their ordinal numbers, and operations with ordinal numbers. At the close are some remarks on concepts which lead to classical paradoxes (such as the set of all sets which do not contain themselves as elements), and on methods of avoiding these paradoxes.—T. C. HOLYOKE, Northwestern University, Evanston, Illinois.

What Is Going on in Your School?

(*Continued from page 414*)

the enrollments in subjects reported in the last 4 years of high school in 15 or more States in 1948–49 are compared with enrollments in the same subjects in 1933–34—the last previous year for which national data are available (Table 2). A complete report of the findings of the present survey is scheduled for publication in the near future.

In February 1949, questionnaires were mailed to all public secondary day schools (except the evening and ungraded) enrolling 500 or more pupils, and to one-half of the schools enrolling fewer than 500 pupils. In all, 3,615 large secondary schools and 10,134 of the smaller secondary schools were circularized. Usable returns were received from 91.8 per cent of the larger schools and from 75.1 per cent of the smaller. This represents a 79.5 per cent response from all schools to which the questionnaire was sent.

The enrollments in the field of mathematics and in the field of science exceed 50 per cent of the pupils in secondary schools (Table 1). Elementary algebra enrolls a greater per cent than any other single subject in mathematics. Among the science courses the largest percentage enrollments occur in general science, biology, chemistry, and physics in that order.

By making appropriate adjustments for sampling and non-response, the 1948–49 survey presents data for *all* public secondary day schools in continental United States. The 1933–34 study, on the other hand, was based on replies from 17,362 secondary schools or approximately 70.2 per cent of the public secondary schools in the United States at that time.

Devices

(*Continued from page 417*)

Holes should be drilled in bar BH at B, S, and A so that $BA=10''$ and $AS=3\frac{1}{2}''$. Bar AD needs holes at A and R; AR must be $3\frac{1}{2}''$ from center to center. Bars OR and OS must each have holes drilled so that $OR=OS=3''$. The only hole needed in bar BM is at B. All holes should be large enough to accommodate $\frac{1}{8}''$ brass bolts. The slot in bar AE need not approach A nearer than $1\frac{1}{2}''$, nor need it be more than $5\frac{1}{2}''$ long. The device can be assembled by inserting $\frac{1}{8}''$ brass bolts at B, A, S, R, and O, and securing them loosely with washers and nuts.

If bar BM is marked off into half inches or quarter inches, the meaning of harmonic division for segment BC can be demonstrated concretely for all positions of C and the corresponding positions of Q and P along BM. It is of interest that when $BQ=QC$, then AQ is perpendicular to BC and AF is parallel to BM. For this position of Q the ratio of internal division is unity ($BQ/QC=1$), and the external division point is the point at infinity.

REFERENCES FOR MATHEMATICS TEACHERS

Edited by William L. Schaaf
Department of Education, Brooklyn College, Brooklyn, N. Y.

Laboratory Mathematics

A prominent development during the past few years has been a lively interest in laboratory methods, involving the use of experiments, models, instruments and a variety of equipment. In this connection, the Eighteenth Yearbook (1945) of the National Council of Teachers of Mathematics contains a wealth of source material and bibliographic data. The purpose of the following notes is to supplement this data and bring it up to date with a minimum of repetition. References to visual aids, in the sense of pictures, charts, slides, film strips, motion pictures, etc., have been deliberately excluded.

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(Continued on page 428)

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
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The Question: What is the place of "mental arithmetic" in the modern teaching of arithmetic?

The Study: Boulware, C. Elwood. *The Emerging Concept of Mental Arithmetic.* Teachers College, Columbia University. 1950.

The purpose of this study was "to show the emerging concept of the meaning and place of mental arithmetic in computation and problem-solving and to consider its teaching possibilities as an instrument of mental growth and development" (p. 19). From an analysis of recent textbooks and professional literature Dr. Boulware decided that "mental arithmetic consists of the various processes by which related fundamental operations in arithmetic are performed by use of the nature of the number system and without the use of algorisms" (p. 7).

In the course of his investigations the author examined seventy-five textbooks printed in the nineteenth and twentieth centuries. He discovered a kind of cyclic trend. Before 1821 the emphasis was on mechanical performance of computations; from 1821 to about 1871 or 1881 he found the stress was on so-called intellectual arithmetic. From about 1891 to 1925 the texts swung back to the operational emphasis. Since 1930 or 1935 the amount of attention given to the development of meaning and understanding in the learning of arithmetic has reflected again a sort of intellectual emphasis.

Dr. Boulware noted that the modern teaching of arithmetic had been influenced considerably by the twentieth century development of the psychology of learning. The conception of the learning process as first involving the perception of gross wholes followed by a stage of differentiation in which refinement and increased clarity of insight appeared was felt to have its counterparts in that arithmetic pedagogy which dealt with grouping and regrouping instead of counting, which sought the development of general ideas instead of special skills and which called for wise judgments, the seeing of relationships and the weighing of values. The use of relational thinking was illustrated in the case of operations by introducing the notions of commutability, associativeness and distributiveness; in the case of number relationships per se the discovery of 9×5 from 10×5 was given as an example.

The new psychology not only presented the notions of gross wholes in the initial stages of learning followed by differentiation but also included the idea of integration. The most important instance of this was the use of ten as the base of a place-value number system; this concept permeated all of the operations on the natural numbers. A second instance was in the various forms which unity could take especially in dealing with fractions.

Finally, a synthesis of the notions of mental arithmetic and the newer learning emphasis was portrayed through five case studies in which students reasoned through problems in their own way, demonstrating unusual original perception of arithmetical relationships through a mental arithmetic approach and revealing the implementation of the newer psychology of learning.

The Question: Do teachers of mathematics approve and practice pupil-centered methods of teaching?

The Study: Syer, Henry W. *Pupil-Cen-*

tered Methods of Teaching Mathematics. Harvard University. June 1950.

During the past twenty-five years the educational spotlight has increasingly played on the learner as a participant and active doer. At least, Dr. Syer found that leaders in education writing in general books on education, in treatises on methods of teaching mathematics, in reports of commissions and committees and in research studies had described approvingly the use of pupil-centered methods. These procedures emphasized sense perception and experimentation in learning mathematics, demanded that concrete experiences precede verbal generalizations, preached that verbalism was not learning, pointed out that physical activity went with mental activity, stated that learning took place in social situations and that methods should be adapted to individual differences. The principal purpose of Dr. Syer's study was to determine whether these ideas were accepted and practiced by the classroom teacher of mathematics.

In gathering his data for grades seven through twelve the investigator sent a questionnaire to 784 of the nations outstanding schools selected with the aid of several organizations considered competent in making the necessary judgments. Both teachers of mathematics and administrators were reached. In the New England area 72 classes were visited and 354 pupils and 75 teachers of this section of the country were interviewed. As part of the study 61 research studies comparing pupil-centered with conventional methods were examined and analyzed for their findings.

Findings

Noting that the methods known as the lecture, recitation, discussion, supervised study and laboratory reflected, in that order, an increasing degree of student participation Dr. Syer found from the questionnaire, to which approximately sixty per cent of the schools responded, that about eighty-six per cent of class time was spent in lecture and recitation, about ten per cent in pupil-centered activities and about six per cent in clerical matters. In the case of the classroom observations the discussion method consumed about four per cent of the class time, the laboratory method less than one per cent and supervised study three to fourteen per cent depending on the definition of the term. From the interviews with the teachers Dr. Syer found that the teachers felt that the pupil-centered methods had value but that the need to change the methods in vogue was not pressing. The teachers reported that the principal sources of their methods were their colleagues and "experience." Methods courses ranked third as a source. Among the factors impeding change the teachers named, in order, the teaching situation, themselves and the pupils. Fifty-two out of seventy-five did not feel that they were restricted in their freedom to vary their methods. Size of class was a dominant element in the complaint about the teaching situation. The pupils who were interviewed were aware that the methods used needed improvement but their suggestions did not veer far from the climate of method that they had contacted.

The sixty-one research studies yielded two hundred seventy-nine measures of outcomes. One hundred fifty of these favored the pupil-centered methods, sixty the conventional and sixty-nine showed no statistically significant advantage. Dr. Syer found that the quality of the research varied widely; in fact, he felt that he could classify only twenty of the studies as excellent or good. Omission of important data, failure to match the groups being compared and the use of instruments of unknown reliability and validity were common.

Finally, by use of chi square techniques the investigator sought to find signs that might reveal where a school stood with respect to pupil-centered methods. Disclaiming any causal implications and suggesting that further investigation was

needed Dr. Syer listed fifteen descriptive factors and fifty-three methods factors as possible indicators. Among the former were the section of the country, attitude toward discussing methods, the budget of the mathematics department and importance attached to individual differences in learning rates. Among the latter the questionnaire data analysis revealed the presence of a mathematics collection, the formation of a mathematics club, the use of mathematics plays and exhibits and the practice of making community surveys of the uses of mathematics as guideposts to the kinds of methods used.

Contemplating the results of the study the reader will either be swept with pessimism or stirred by the challenge depending on his personal characteristics.

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ATTENDANCE RECORD OF THE TWENTY-NINTH ANNUAL MEETING

Hotel William Penn, Pittsburgh, Pa.
March 28-31, 1951

Name of State	No.	Name of State	No.	Name of State	No.
Alabama	3	Kentucky	1	Oklahoma	2
Arkansas	1	Louisiana	2	Pennsylvania	624
California	1	Maryland	17	Rhode Island	2
Colorado	2	Massachusetts	16	South Carolina	2
Connecticut	4	Michigan	20	Tennessee	8
Delaware	2	Minnesota	1	Texas	2
District of Columbia	14	Missouri	3	Virginia	14
Florida	5	Nebraska	2	West Virginia	27
Georgia	4	New Hampshire	3	Wisconsin	9
Illinois	23	New Jersey	21	Canada	8
Indiana	13	New York	66		
Iowa	5	North Carolina	4	35 States and Canada	
Kansas	1	Ohio	96	Total	1028

AIDS TO TEACHING

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Public Affairs Pamphlets, 22 East 48th Street, New York 16, N. Y.

Booklets; $5\frac{1}{2}'' \times 8\frac{1}{2}''$; 32 pages each; 1–9 copies are $.20 each, discounts on larger quantities.

Description of B. 63: This booklet, by LeBaron R. Foster, discusses the various sources of consumer credit, how to use credit to advantage, how to tell the rate you are paying, why credit is expensive, and outlines a program for social protection. There is a summary of warnings at the end and an excellent bibliography.

Description of B. 64: This publication by William T. Foster describes the devious ways used by small loan companies to charge exorbitant interest rates for small loans. It describes the various ways in which loan sharks operate to evade the law and to prevent effective laws from being passed. The effect of these illegitimate business enterprises is contrasted with the beneficial results of enforceable laws. It concludes its message by giving a series of specific suggestions on how to avoid being "duped" by a loan shark. A bibliography for further reading is included.

Appraisal of B. 63 and B. 64: These descriptions are more general than some booklets concerning credit and will appeal to a more mature audience. Although such consumer mathematics is more prevalent in junior high school courses, these booklets are written for the senior high school and adult level. Since general mathematics is appearing in grades ten, eleven and twelve, they should be very helpful. By using the facts outlined in these booklets, pupils can investigate the loan practices of each local community, with advantage to both themselves and the community.

B. 64—Know Your Money

U.S. Secret Service, Treasury Department, Washington, D. C.

Booklet; $6'' \times 9''$; 32 pages; Free.

Description: This useful booklet is filled with many extremely interesting pictures. It is based upon the idea that "the best check to any evil is always that provided by an educated public." By telling the public how to detect counterfeit money and forged government checks, the Secret Service hopes to cut down such illegal practices. There is first a short history of money and description of our paper currency. Throughout there are excellent enlarged pictures of good and of counterfeit money. Then there are stories from the Secret Service files; a description of what to do when money burns or wears out; the scientific side of detection; how to detect counterfeit coins; and a section on government checks.

Appraisal: This book makes interesting reading even disregarding its educational value. Certainly copies should be around for supplementary reading. There is no reason why time could not be spent on a very interesting unit on counterfeit money with talks by bank experts, displays of counterfeit money, contests to see who can detect the difference in specimens displayed. The old cry that this is not in the

curriculum of the mathematics department will not deter those who see the usefulness of this topic and who see that the justification is just as strong in mathematics as in some other subjects. If social studies or science teachers are already doing this, it may be left alone, but if not we should reach for some of the lively topics to brighten our work. The classes will pounce on it.

B. 65—What Everybody Ought to Know about this Stock and Bond Business

Merrill Lynch, Pierce, Fenner & Beane, 70 Pine Street, New York 5, N. Y.

Pamphlet; $8\frac{1}{2}'' \times 11''$; 6 pages; Free.

Description: This pamphlet was written to answer the most common questions asked stock brokers about stocks and bonds. It answers questions such as: What are stocks and bonds? Why should anybody buy stocks? Who may buy stocks and bonds? When should you buy stocks and bonds? How do you buy and sell stocks and bonds? What are the different kinds of stocks? What are the factors that affect the prices of stock on the stock market? Its main objective is to promote the careful purchase or sale of stocks and bonds for investment purposes. It suggests that this investment be made in an understanding manner with surplus funds that are not needed for necessary expenditures, insurance or savings sufficient to meet emergencies.

Appraisal: This pamphlet is written in a simple, concise manner so that it is readable to the man on the street. Thus, it is of an appropriate level for junior or senior high school students. Although it is written to promote the sale of stocks and bonds, it emphasizes the need for caution, such as "—no one should speculate unless he can afford to take risks" or "investigate —then invest" or "—not everybody should buy stocks and bonds." It also emphasizes the purchase of stocks and bonds for investment purposes rather than for speculation. It does not include any mathematical problems, but does use many terms and figures such as par value and stock market quotations.

B. 66—Vernier Tools

Brown & Sharpe Manufacturing Co, Providence 1, R. I.

Pamphlet; $6'' \times 9''$; 3 pages; Free.

Description: This brief pamphlet was written to teach one how to read a vernier. After describing the principle of the vernier, it gives sample problems with drawings to illustrate how to read the vernier. The examples show why the vernier reading gives a certain decimal value. The examples include English and metric units of measure. Additional instructions are given on how to operate vernier tools to attain maximum accuracy.

Appraisal: This pamphlet should be used with a large model of a vernier to give instruction in measurement and the reading of a vernier. The print is small and the examples somewhat complex for immature high school students. However, the teacher will find it valuable in showing how to teach the principle of the vernier.

EQUIPMENT

E. 43—Arithmetic Readiness Cards: Grouping

Scott, Foresman and Company, 114 East 23d Street, New York 10, N. Y.

Set of cards; $6\frac{1}{2}'' \times 8\frac{1}{2}''$; number cutouts; teacher's guidebook; in box; \$3.20 (educational discount).

E. 44—Arithmetic Readiness Cards: Number System

Scott, Foresman and Company; 114 East 23d Street; New York 10, N. Y.

Set of cards; $6\frac{1}{2}'' \times 8\frac{1}{2}''$; number cutouts; tabulation cutouts; teacher's guidebook; in box; \$3.20 (educational discount).

Description of E. 43: This set contains 54 tagboard cards with printed pictures on both sides, 9 diecut cards which are separated into 81 cutouts each printed

with a number from 2 to 10, and a teacher's guidebook. The pictures are numbered 1 to 108. Those numbered from 1 through 54 show organized groups of objects arranged in smaller sub-groups, whereas those numbered from 55 through 108 show objects in unorganized groups. Each card has a cut corner which aids in arranging the cards so that all pictures or organized groups or all pictures of unorganized groups will be face up. Each card also has two slots into which a number cutout can be inserted. These cards aim to supply the child with pictorial experiences in the transfer from recognition of small number groups to the understanding of abstract numbers.

Description of E. 44: This set contains 54 tagboard cards with printed pictures on both sides, 9 diecut cards which are separated into 135 cutout pictures of sticks or tally marks, 9 diecut cards which are separated into 135 cutouts each printed with a number digit 0 to 9, and a teacher's guidebook. The pictures are numbered from 1 to 108. Those numbered from 1 through 54 show organized groups of objects arranged in sub-groups of ten. These sub-groups are arranged in pyramid formation to facilitate identification of a group of ten without counting. Pictures numbered from 55 through 108 are arranged in a similar manner except that groups of ten are shown as bags, bundles, boxes, etc., which the child is required to accept as a group of ten. Each card has a cut corner which aids in arranging the cards so that all groups employing pyramid formation will be face up. Each card also has four slots, into which two tabulation cut-outs or two number cutouts may be inserted, with the exception of 9 cards which have six slots which allow three insertions. These cards aim to supply pictorial material useful in developing the basic concepts of our number system.

Appraisal of E. 43 and E. 44: These sets of cards provide attractive material useful in developing the concepts and relationships of abstract numbers. They may be used independently or correlated with the common procedure of employing disks, sticks and other objects. They are appropriate for teaching methods which allow the pupils to discover the ideas involved in grouping and in our number system.

The cards appear to be reasonably durable. They are soiled easily, but can be cleaned readily with a gum eraser. The variety of brightly colored drawings of toys, animals and similar objects should be familiar to most children in the early grades. (Reviewed by Robert H. Wyllie, Boston University, Boston, Mass.)

E. 45—Experimenting with Numbers

Houghton Mifflin Company, 2 Park Street, Boston 7, Mass.

Number Kit (See below); Designed by Catherine Stern; $24.00 net to teachers and schools.

Description: A set of materials that represents a new approach to number readiness. The box of materials contains:

1 Counting Board with Number Guide
1 Unit Box filled with Unit Blocks (two each of Block 1–9; one 10-Block)
1 Unit Box filled with 100 Cubes
1 Set of 10 Number Cases
1 Extra Set of 10 Unit Blocks
2 Sets of 10 Number Markers
1 Zero, 1 Plus Sign, 1 Equal Sign
A Teacher's Manual

The Counting Board contains 10 vertical grooves into which blocks of the corresponding size may be inserted. Each block is scored to show its cube-units: the 1-block is 1 unit long, the 2-block 2 units, the 3-block 3 units and so on up to the 10-block which is 10 units long. Each block is a different color. These Unit Blocks are used to represent the number range from 1 to 10. The grooves in the counting board are marked like the blocks. At the top of each groove is a removable marker which has the corresponding number symbol on it.

The Pattern Boards contain from 1 to 10 blanks in two columns. These boards are made to receive the single cubes. The first board contains 1 blank, the second

has 2 and the last one has 10 blanks. These boards may be used to teach patterns of certain configurations. The pattern of one number is related to the pattern that precedes and follows it. The Pattern Boards give meaning to odd and even numbers and aid in the mastery of some addition and subtraction facts.

The Unit Box is a case 10 units square. It is used to hold the block pairs such as 2 and 8, 6 and 4, etc., that tell the story of 10. The Number Cases from 1–9 are also square. These cases help the pupil in discovering the block pairs (combinations) that fill each case.

The Number Markers 1 to 10 and symbols for plus, equal, and zero are used to record the addition facts that the children discover in the number cases. The oral addition stories can be recorded in this manner before the actual writing of numbers is taught.

Appraisal #1: Experimenting with Numbers is the beginner's course in Structural Arithmetic which is based on measuring rather than counting. (This method is described fully by the author in her book *Children Discover Arithmetic*, Harper and Brothers, 1949). When a child adds a 3-block to a 4-block, he sees that the two blocks can be equal only to the length of the 7-block.

The Teacher's Manual contains a carefully developed sequence of demonstrations, experiments, and games that enable the child to discover number relationships for himself. On the first level, the children make experiments without using the number names or their written symbols. On the second level the names of the numbers are introduced and the children learn the expressions and meaning of oral addition. On the third level, the number symbols from 1 to 10 are taught and the children learn to let numbers stand for blocks and patterns. The Number Markers and symbols may now be used to express the addition facts that the children discover.

Although this set of materials is largely used in the kindergarten and first grade, the aids could be used to great advantage by the second and third grade teachers in getting mastery of the primary addition and subtraction facts.

Firsthand experiences with the blocks, cubes, and patterns which are true representations of the numbers, lead to learning by insight which provides an understanding of numbers that can never be obtained by rote memorization or drill. (Reviewed by Mrs. Ida Mae Heard, Southwestern Louisiana Institute, Lafayette, La.)

Appraisal #2: This material is so functional in purpose that one should not approach a developmental number concept curriculum to provide individualized instruction for the child without this material. Qualities, which stimulate interest for exploring to discover number relationships for himself, promote mastery of Number Concepts by being able to progress from the use of concrete materials with attractive colors, interesting shapes, convenient sizes particularly adaptable to comparing varying number groups; to the semi-abstract, by using number names to develop familiarity with expressions and meanings of the oral concept; and by using number symbols which leads to the expression of the number concept, thus providing for three levels to make every step in his learning meaningful. Structural Arithmetic proves to be a means of completely eliminating remedial cases, in that the devices are self-corrective, motivating the child by experiencing success to the extent of his potential ability to meet the level of perception of his maturation. Making implications of the instructional program of Structural Arithmetic within the classroom, promotes the child's progress in continuous developmental growth in Number Concept through better guidance and observation techniques. (Reviewed by Alice Funfar, University of Minnesota, Minneapolis, Minn.)

FILMS

F. 59—Motivating the Class

McGraw-Hill Publishing Co., 330 West

42nd Street, New York, N. Y.

Film; 16 mm.; Black and white; Sound; 2 Reels.

Description: This film shows a student teacher teaching a mathematics class. His first lesson is a lecture on the nature and development of mathematics and is a failure due to lack of interest by the students. In a conference with his supervising critic teacher, suggestions for meeting the varied goals of the class members are discussed. Shots of subsequent lessons show the student teacher using practical applications, field work, motion pictures, projects, graphs of progress and posters to stimulate the successful study and learning of mathematical principles.

Appraisal: As in most teaching films, this one must cover a vast topic in a short twenty-minute portrayal. However, it succeeds in giving the viewer many practical ideas for adding interest and meaning to his instruction. In order to do this in such a short time some exaggerated and unrealistic situations are used. Although the goals of students are the prime motivators of learning, and are discussed by the supervising teacher, the subsequent development shows very little of this intrinsic motivation. However, the film is effective in showing how instruction is improved and interest developed by the use of a variety of materials. The selection of a practice teacher to illustrate a poor lesson and a changed approach is very appropriate. This is a useful film for the training of teachers in any field and is most usable in the training of teachers of mathematics.

F. 60—A Thousand Hours

Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Mich.

16 mm. film; Black and white; Sound; 10 minutes; 1 reel; $36.50.

Description: The opening scene shows a boy coming out of school and entering a clubhouse which has a sign on it "Invention Shop." Two other boys are in the shop working. A discussion follows concerning the value of mathematics.

The captain of one of the great airline transports enters. The boy that had been seen coming out of school expresses his desire to quit school. The captain inspires the boys with an interesting explanation of the background of the sciences necessary to pilot training. One of the captain's trips is used in this explanation.

The following are a few of the subjects the captain covers: advanced geography used in looking up weather reports; foundation of physics used in looking over plan of flight; radio used for the flight officer's report while flying; and trigonometry and calculus used in making the reports.

A change in the boys' attitudes is noticeable as the film ends.

Appraisal: Twenty-two teachers were together to evaluate this film. Considering the entire film they found that it can be most effectively used as a motivating device. Thirty-nine per cent rated the film as good when used to introduce new material, but 22% said that it was poor for this same purpose. The same situation existed when they considered using the motion picture to augment explanations. The teachers believed this film good for providing a common experience but it definitely is of no use in developing skills or for review purposes. The previewers checked the grade level as usable from the 7th to the college grades, with marked predominance in the 8th, 9th, and 10th. Also, it can be used in all mathematics courses and especially in algebra. Over 90% scored the speed of development of ideas and duration of scenes as moderate. Three of the evaluators found some inaccuracies in the mathematical content while the remainder found none. Just under 60% viewed the film as completely holding the interest of students while "partially" was pencilled by the others. Given three alternatives for rating the teaching methods in the film as conducive to learning, they scored 32% as "completely," 63% as "partially" and 5% as "not at all."

The photography was good. The sound of the movie was given 9 for poor, 5 for fair, 7 for good and 1 for very good. The effectiveness of characterizations and background objects was positively good. There was complete agreement that the presentation would encourage further pupil activity in mathematics and in other subjects. All checked the captions as being of appropriate length and pertinent. The dialogue was definitely effective. The arrangement of topics was in a logical sequence and the amount of material is reasonable for comprehension in one showing. All said that there was coordination between sound and picture. Of the entire group, 71% did not believe that the content could be just as effectively and efficiently presented in some other way. The teachers agreed that the film attempted to supplement rather than replace the instructor. Nineteen remarked that they would use this film. Some comments expressed were, "This film is good for motivating further pupil activity in mathematics"; "It will also have application as an occupational guidance film." (Reviewed by Anthony di Luna, Raymond Fleet and Kenneth Hatheway.)

INSTRUMENTS

I. 32—Blackboard Compass

Milton Bradley Company, Springfield 2, Mass.

Compass; No. 8076; 16″ legs; $2.00.

Description: The compass is made of two varnished wooden legs which are connected at one end by a wing screw which enables the user to fix the setting for drawing circles with desired radii. One leg of the instrument contains a tiny rubber tip whose function is to prevent the compass from sliding while circles are being drawn. The other leg contains an opening for chalk and a sliding brass ring is used to hold the chalk firmly in place.

This excellent compass has been purchased by many schools because it is both simple and durable.

Appraisal: The blackboard compass is a valuable instrument in any mathematics course which includes the study of circles. The price of this instrument has increased greatly in the past ten years, but is worth $2.00 nevertheless. (Reviewed by Bernard Singer, Hyannis, Mass.)

PICTURES

P. 11—Portraits

Yerkes Observatory, Williams Bay, Wis.

Pictures; see below.

Description: In 1903, the Yerkes Observatory of the University of Chicago started to reproduce lantern slides, transparencies, and prints of astronomical phenomena. Their collection includes negatives of astronomers who were also notable mathematicians. Portraits of the men listed below are available in several forms —$3\frac{1}{4}''\times 4''$ glass slides, $8''\times 10''$ unmounted prints, and $8''\times 10''$ window transparencies. The price per slide is $.75, $.50 per print, and $2.00 per transparency.

F. 8	Tycho Brahe	*F. 42*	Laplace
F. 17	Albert Einstein	*F. 50*	Newton
F. 23	Galileo	*F. 53*	Poincare
F. 24	Gauss		

Appraisal: Since the photographs are to be used for the creation of a mathematical atmosphere, the prints are superior to the slides which cannot be projected for a long period of time and the transparencies which are too expensive. Discussions of the mathematical significance of these men will have to be brief since students of secondary mathematics are hardly in the position to appreciate specific advantages of higher mathematics. The prices of the slides, prints, and transparencies are all within reason but the prints serve the desired purposes and are relatively inexpensive. (Reviewed by Bernard Singer, Hyannis, Mass.)

LETTERS

Order of Operations

Miss Edith Wood of Anchorage, Kentucky, calls attention in the February, 1951 issue of THE MATHEMATICS TEACHER to a difference among authors about the order of operations, addition, subtraction, multiplication, and division. Of course there is no disagreement if proper symbols of aggregation are used.

The difference occurs in a problem like $12+6\div 2\times 3$. There is general agreement that multiplication and division should be performed first, and then addition and subtraction. One group of authors say that, where there is multiplication and division, the multiplication should be first performed and then the division. This group would say that the correct answer to the problem $12+6\div 2\times 3$ is $12+6\div 6$, or 13. The other group would say that $12+6\div 2\times 3 = 12+3\times 3 = 12+9 = 21$. Of course if proper signs of aggregation are used, the question becomes entirely academic.

Chrystal* in his *Algebra*, does not agree with the first group. He says on page 14 "$\times a\div b\times b = \times a$ and $\times a\times b\div b = \times a$." Some authors would have the first of these equal to $\frac{a}{b}$ and the second equal to a.

Chrystal says again: "Another notation for a quotient (other than $a\div b$) is sometimes used, namely, $\frac{a}{b}$ or a/b. As this is the notation for fractions, and therefore has a meaning already attached to it in the case where a and b are integers, it is incumbent upon us to justify its use in another meaning. To do this we have simply to remark that b times $\frac{a}{b}$, that is, b times a of the bth parts of unity, is evidently a times unity, that is, a; also, by the definition of $a\div b$, b times $a\div b$ is a. Hence we conclude that $\frac{a}{b}$ is operationally equivalent to $a\div b$ in the case where a and b are integers."

Again on page 15, in establishing the law of commutation he shows that

$$a\times b\div c = a\div c\times b = \frac{ab}{c}\cdot$$

Others would have

$$a\times b\div c = \frac{ab}{c} \quad \text{and} \quad a\div c\times b = \frac{a}{bc}\cdot$$

A difficulty arises in a problem like $a\div ab$. Probably both groups would give the answer to this as b. If, however, the same expression is written as $a\div a\times b$, there is a disagreement which cannot be settled by any rule about order of operations. It seems evident that the form ab is not exactly equivalent to the form $a\times b$. ab has the force of a parenthesis, that is, the multiplication must be performed before other operations are. This is not true of $a\times b$.

A similar situation arises with the form $\frac{a}{b}\cdot$

Thus, probably both groups would say that $a\div\frac{a}{b}$ is equal to b.

But Chrystal, on page 15, in establishing the law of commutation, says that $a\div a\div b$ is equal to $\frac{1}{b}\cdot$ Probably both groups would agree with this result.

Thus, despite Chrystal, ab is not entirely equivalent to $a\times b$ nor is $\frac{a}{b}$ equivalent to $a\div b$ when there are continued operations.

A statement which correctly covers all cases is this:

I. Operations enclosed in symbols of aggregation, either expressed or implied, (as ab, $\frac{a}{b}$, $\sqrt{}$, ( ), [ ], { }, or the vinculum) should first be performed.

II. Multiplications and divisions should then be performed in the order in which they occur from left to right.

III. Finally additions and subtractions in any order.

VIRGIL S. MALLORY
Head, Department of Mathematics
State Teachers College
Montclair, N. J.

* G. A. Chrystal, *Algebra, An Elementary Textbook for the Higher Classes of Secondary Schools and for Colleges* Part I, Fifth Edition. Adams and Charles Black, London, 1910.

Report on the Second Delegate Assembly

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups
University of Wisconsin, Madison, Wisconsin

THE AGENDA of the Second Delegate Assembly of The National Council of Teachers of Mathematics, held in Pittsburgh, March 29 and 30, 1951, was determined by questions proposed by Affiliated Groups during the year and reported in this section of THE MATHEMATICS TEACHER and in the *Newsletter of the Affiliated Groups*. Among the topics given consideration by the Delegates were clarification of requirements for affiliation, relationships between Affiliated Groups and State Representatives, distribution and sharing of materials prepared by the various Groups, problems of publicity, Speakers Bureau, budget of the Committee on Affiliated Groups, possibilities for a traveling exhibit, and contest sponsorship.

The program of the Assembly also included a welcome to the Delegates by President Harry Charlesworth, a ceremony in recognition of the eleven Groups affiliated since the First Delegate Assembly in Chicago in 1950, the annual report of the Committee on Affiliated Groups, and reports on activities of Affiliated Groups of the National Council for the Social Studies and the National Council of Teachers of English.

All recommendations of the Second Delegate Assembly were subsequently approved by the Board of Directors.

Copies of the Minutes of the Second Delegate Assembly will be sent upon request addressed to John R. Mayor, North Hall, Madison 6, Wisconsin.

REQUIREMENTS FOR AFFILIATION

The Second Delegate Assembly, interpreting the actions of the First Delegate Assembly[1], voted that it be required that each Affiliated Group:

1. Have a written constitution and by-laws.
2. Maintain a list of its members with teaching positions and addresses and with indication of members of the National Council, and file a copy of the list with the Chairman of the Committee on Affiliated Groups.
3. Renew its affiliation annually.
4. Renew affiliation and pay annual dues each year between October 1 and December 31.
5. Pay annual dues to the National Council under the plan

   less than 150 members $3.00
   151 to 250 members $4.00
   251 to 350 members $5.00
   over 350 members $6.00

   except that annual dues are waived for groups with 75% or more of their members also members of the National Council, and new Groups are not required to pay affiliation dues during the school year in which they become affiliated.

Point 5 implies that, at least for the present, no minimum has been set for the number of members in an Affiliated Group.

DELEGATE ASSEMBLY

In regard to the Delegate Assemblies it was decided that:

1. One vote be allowed each Group regardless of size.
2. No Delegate be allowed to vote at a Delegate Assembly if the annual dues for renewal of affiliation have not been paid.

SHARING OF TEACHING AIDS

The Delegates voted that:

Copies of all printed and duplicated materials of each Group be sent to the Chairman of the Committee on Affiliated Groups and to the Washington office of the National Council.

FUNDS

It was ruled that the expenditure of funds collected from the Groups through affiliation dues be left to the chairman of the Committee on Affiliated Groups. The discussion revealed that the Groups favored provision for financial support for a Speakers Bureau, and for the establishment of a travelling exhibit, and, if necessary because of cost, reduction in the number of issues of the *Newsletter*.

[1] John R. Mayor, "The National Council and Its Affiliated Groups," THE MATHEMATICS TEACHER, XLIII, 345–347.

Future Plans

While no official action was taken on other topics on the agenda the Assembly received with favor proposals for better Affiliated Groups and National Council publicity, early establishment of a Speakers Bureau as a service agency to Affiliated Groups, establishment of a traveling exhibit for meetings of Affiliated Groups and possible use by schools, encouragement of mathematics contest sponsorship, and an annual luncheon for Delegates of Affiliated Groups.

New Groups

Recognition was given to the following eleven groups, which completed affiliation since the First Delegate Assembly:

The Colorado Council of Teachers of Mathematics
Affiliated May 27, 1950; Charter no. 84.
Mathematics Department, Alabama Education Association
First Affiliated April 10, 1936; Charter no. 27. Renewed July 8, 1950.
Oklahoma City Mathematics Council
First Affiliated April 12, 1936; Charter no. 28. Renewed November 1, 1950.
Arkansas Council of Teachers of Mathematics
Mathematics Section, Arkansas Education Association, Affiliated November 3, 1939; Charter no. 48. Renewed under new name, December 13, 1950.
Pennsylvania Council of Teachers of Mathematics
Affiliated February 11, 1951; Charter no. 85.
Pinellas County (Florida) Council of Teachers of Mathematics
Affiliation renewed February 12, 1951; Charter no. 86. (date of earlier affiliation not determined)
Association of Teachers of Mathematics of Philadelphia and Vicinity
Affiliated February 19, 1951; Charter no. 87.
Mathematics Section of South Carolina Education Association
Affiliated May 15, 1939; Charter no. 60. Renewed March 16, 1951.
Mathematics Section, South Dakota Education Association
Affiliated March 16, 1951; Charter no. 88.
Benjamin Banneker Club of Washington, D. C.
Affiliated April 3, 1938; Charter no. 42. Renewed March 24, 1951.
Utah Council of Teachers of Mathematics
Affiliated March 24, 1951; Charter no. 89.

Official Delegate List

Mary C. Rogers and John R. Mayor served as co-chairmen of the Second Delegate Assembly. Elaine Rapp was secretary of the Assembly with Alice M. Reeve as assistant secretary. Special reports were presented by Jackson B. Adkins, Exeter, New Hampshire; Margaret A. Striegl, Wauwatosa, Wisconsin; Mary C. Rogers, Westfield, New Jersey; Madeline Messner, Roselle, New Jersey; Houston Karnes, Baton Rouge, Louisiana; Kenneth Brown, Knoxville, Tennessee.

The official Delegates were:

Mathematics Department, Alabama Education Association
Mrs. E. L. Asbury, Holt
Arkansas Council of Teachers of Mathematics
R. W. Young, Arkadelphia
California Mathematics Council
Charles C. Fabing, Los Angeles
The Colorado Council of Teachers of Mathematics
Harry W. Charlesworth, Washington and Denver
Mathematics Section, Eastern Division, Colorado Education Association
Lillian M. Sullivan, Denver
Florida Council of Teachers of Mathematics
W. A. Gager, Gainesville
Dade County Council of Teachers of Mathematics
Verna Kimler, Miami
Hillsborough County Council of Mathematics Teachers
Howard L. Gallant, Tampa
Georgia Council of Teachers of Mathematics
Gladys M. Thomason, Sandersville
Bess Patton, Atlanta (Alternate Delegate)
Illinois Council of Teachers of Mathematics
Henry Swain, Winnetka
Chicago Elementary Teachers' Mathematics Club
Joseph J. Urbancek, Evanston
Men's Mathematics Club of Chicago and Vicinity
M. D. Oestreicher, Chicago
Women's Mathematics Club of Chicago and Vicinity
Martha Hildebrandt, Maywood
Indiana Council of Teachers of Mathematics
K. Eileen Beckett, Lebanon
Iowa Association of Mathematics Teachers
Henry Van Engen, Cedar Falls
Kansas Association of Teachers of Mathematics
Gilbert Ulmer, Lawrence
Kentucky Council of Mathematics Teachers
Edith Wood, Anchorage
Louisiana-Mississippi Branch of The NCTM
Houston T. Karnes, Baton Rouge
Mathematics Section of the Maryland State Teachers Association
Margaret L. Heinzerling, Baltimore
Detroit Mathematics Club
William Slemer, Detroit

Minnesota Council of Teachers of Mathematics
Florence Collins, St. Paul
The Association of Teachers of Mathematics in New England
Jackson B. Adkins, Exeter, New Hampshire
Association of Mathematics Teachers of New Jersey
Mary C. Rogers, Westfield
Association of Teachers of Mathematics of New York City
Barnett Rich, New York
Nassau County Mathematics Teachers' Association
Elaine Rapp, Freeport, New York
Alice M. Reeve, Rockville Centre (Alternate Delegate)
Suffolk County Mathematics Teachers Association
Edmund Miles, Amityville, Long Island, New York
Greater Cleveland Mathematics Club
Charles E. Scott, Cleveland
Oklahoma Council of Teachers of Mathematics
James H. Zant, Stillwater
Tulsa Mathematics Council
Muriel Lackey, Tulsa
Ontario Association of Teachers of Mathematics and Physics
Robert E. K. Rourke, Newmarket
Pennsylvania Council of Teachers of Mathematics
Lee E. Boyer, Millersville
Mathematics Teachers Association of Western Pennsylvania
L. McClure Lanning, Pittsburgh
The Association of Teachers of Mathematics of Philadelphia and Vicinity
M. Albert Linton, Jr., Philadelphia
Mathematics Section, South Carolina Education Association
Margaret W. Huff, Columbia
Mathematics Section, South Dakota Education Association
Joy Hamrin, Sioux Falls
Mathematics Section, East Tennessee Education Association
Kenneth Brown, Knoxville
Mathematics Section, Texas State Teachers Association
Ida May Bernhard, San Marcos
Dallas Elementary Mathematics Association
Ruth Clough, Dallas
Mathematics Section, Virginia Education Association
Burton F. Alexander, Petersburg
Richmond Chapter, The NCTM
Mary E. Hawkins, Richmond
Benjamin Banneker Mathematics Club, Washington, D. C.
Guinevere D. White, Washington
The West Virginia Council of Mathematics Teachers
Frances Grimm, Huntington
Wisconsin Mathematics Council
Margaret A. Striegl, Wauwatosa

Groups Not Yet Affiliated

New York State Council
Alice M. Reeve, Rockville Centre
Ohio Council of Teachers of Mathematics
Ona Kraft, Cleveland

NEWS NOTES

The annual **Convention of the Central Association of Science and Mathematics Teachers** will be held in Cleveland, November 22–24 of this year. The Hollenden Hotel has been selected for headquarters. Interesting general meetings are scheduled for Friday morning and Friday afternoon, November 23, and Saturday morning, November 24. Among the featured general speakers are Dr. Keith Glennan, President of Case Institute of Technology and member of the Atomic Energy Commission; Dr. Elvin C. Stakman, Chief of the Division of Plant Pathology and Botany, University of Minnesota; and Dr. David Dietz, Science Editor, Scripps Howard Newspapers. Dr. Dietz will speak at the banquet Friday evening. Section meetings in Elementary Mathematics, Elementary Science, General Science, Mathematics, Biology, Chemistry and Physics will be held Friday. Group meetings in areas of Elementary Schools, Junior High Schools, Senior High Schools, and Conservation will be held Saturday.

In Quest of Truth is the title of a short historical pageant on mathematics written by Daniel B. Lloyd, of Wilson Teachers College in Washington, D. C. It is suitable for reading or dramatization by junior or senior high school students, for classroom use, and for assembly or club programs. Sample copies may be obtained from the author for 25¢ or in quantities at $1.00 a dozen.

The Treasury Department is distributing to school administrators a 6-page folder entitled DEFENDING AMERICA: SCHOOL SAVINGS IN THE NATIONAL EMERGENCY. This folder restates the basic educational purposes of the Treasury's School Savings Program—to achieve long-range training and practice in personal thrift and the wise use of natural and other resources. The folder also asks for a patriotic expansion of School Savings in this time of national emergency.

MINUTES OF ANNUAL BUSINESS MEETING

National Council of Teachers of Mathematics
Hotel William Penn, Pittsburgh, Pennsylvania

First Session

Thursday, March 29, 1951, 3:30–4:30 p.m.

The meeting was called to order by President H. W. Charlesworth, with 78 people present.

No minutes of the previous meeting were read.

The resignation of Dr. W. D. Reeve as Editor-in-Chief and the recent affiliation with the N.E.A. necessitated the appointment of a new Editor-in-Chief for The Mathematics Teacher, and an Executive Secretary. In order to fill these appointments a revision of the By-Laws is required. The following Committee was appointed to serve for this purpose:

George Hawkins, Lyons Township High School and Junior College, La Grange, Illinois
Mary Potter, Board of Education, Racine, Wisconsin
Marie S. Wilcox, Chairman, George Washington High School, Indianapolis, Indiana.

The proposed By-Laws, as they appeared in the February, 1951 issue of The Mathematics Teacher were presented by Mrs. Wilcox. She called attention to each article proposed, and changes in the old By-Laws were noted with the reasons for changing. At the end of the reading and presentation, Mrs. Wilcox explained that acceptance of the By-Laws would not prohibit minor changes and that the Board of Directors had authorized the continuance of the Committee for one year to consider any suggestions for further revision. Mrs. Wilcox made it clear that letters with suggestions would be welcome. She then moved that the proposed By-Laws as printed in the February issue of The Mathematics Teacher be accepted. The motion was seconded by Dr. R. R. Smith.

The discussion which followed brought forth the following questions:

1. The advisability of limiting the nominee for President to a person who has served on the Board within the preceding five years.
2. The consideration of restricting Board members to one term of three years, with a specified lapse of time before they would be eligible for re-election.
3. The possibility of increasing the size of the Board of Directors.
4. The advisability of having a future president elected for one year as a non-voting member of the Board.

Dr. Rosskopf moved that the By-Laws be tabled. There being no second to this motion, no action was taken.

The vote was then taken on the original motion, which was approved. There was one dissenting vote.

Dr. Christofferson moved that a vote of thanks be extended to the members of the Committee on the Revision of the By-Laws. This was seconded by Dr. R. R. Smith and met with much applause.

Mr. Schreiber gave the results of the recent elections. 1,015 votes were tallied.

Vice-President representing College Mathematics
James H. Zant
Members of the Board of Directors
William A. Gager
Lucy E. Hall
Henry Van Engen

Congratulations were offered to these members and each was introduced.

There being no further business, Mr. Schreiber moved the meeting be adjourned. This motion was duly seconded and passed.

Respectfully submitted,
Agnes Herbert, *Acting Secretary*

Second Session

Saturday, March 31, 1951, 11 a.m.

The meeting was called to order by the President with 125 present.

Announcement of previous Board action on the place for the next two annual meetings was given; Nebraska, Iowa and Atlantic City had all extended invitations. It was decided that the 1952 meeting should be held in Des Moines, Iowa and that the invitation of Atlantic City should be accepted for 1953.

The question was then raised of the position of the National Council in the present National Emergency situation. A Committee has been appointed to formulate a platform and reaffirm some of the objectives in the Commission Report for May, 1945.

Dr. Hildebrandt urged those present to return to their own localities and emphasize two things:

1. Every student should take as much mathematics as possible.
2. Students with special aptitude should take every part of mathematics available in order to be useful in scientific work.

Mr. Lloyd asked that we stand pat on what has been done and the Committee be congratulated on their careful thinking of this program.

Mr. Charlesworth told of the plans on smaller publications and Year Books.

Mr. Christofferson moved the adoption of

the following tribute to Dr. Kenneth Brown, to be published in THE MATHEMATICS TEACHER and a copy of which is to be sent to Mr. Brown:

The National Council of Teachers of Mathematics has become really a national council with a spirit of devoted service to teachers and children because of the unselfish work of many men and women. Conspicuous among these persons has been the man who for several years has been the Chairman of State Representatives. His devotion to the Council, his vision of the importance of the organization, his tireless work in keeping a host of representatives alert and active have been constant influences in building the Council to one of the strongest teacher organizations in the country.

His retirement from this office in the reorganization of the Council will be a serious loss. His influence will continue and his activity in promoting the Council will increase. We express gratitude to Kenneth Brown for his selfless and effective service. We thank him and look forward to his continued influence in the Council.

This was seconded by Mr. Peak and carried.

Mr. Shuster moved the adoption of the following tribute to Mr. Edwin W. Schreiber, to be published in THE MATHEMATICS TEACHER and a copy of which is to be sent to Mr. Schreiber:

During the long and varied life of the National Council of Teachers of Mathematics many Presidents, Vice-Presidents, Editors, and Board Members have played their brief part on the stage and have then passed on to constitute the growing list of former leaders of the National Council. However, during all these long years, one Charter Council Member remained constantly on the Board. He has had charge of the purse strings of the Council all these years and has been a Rock of Gibraltar in resisting any enthusiastic but unwise scheme to deplete the treasury. He has been the pilot who has constantly kept the Constitution of the Council before him like a chart and has steered the Council on a course that has avoided all the legal, diplomatic, and financial rocks and shoals

It is unnecessary to say that this firm, wise, and kindly man who has rendered such long and valuable service to the Council is our Secretary-Treasurer, Edwin W. Schreiber.

We wish continued good health and happiness to him and we hope we shall have him as a regular attendant and contributor to our meetings for many, many years.

This was seconded by Dr. Bakst and carried.

Dr. Bakst moved that we offer our thanks to Dr. Hildebrandt as Editor-in-Chief of THE MATHEMATICS TEACHER for the excellent job that he has done thus far. This was seconded by Mr. Swain and carried.

Mr. David Kotler presented the following resolution, which after the reading was seconded by Mr. T. S. Klein:

WHEREAS, The National Council of Teachers of Mathematics is a Department of the National Education Association, and

WHEREAS, The National Education Association adopted at its summer 1950 meeting held in St. Louis the following resolution: "Members of the Communist Party shall not be employed in our schools. Communist organizations and communist front organizations should be required by law to register with the Attorney General of the United States." And

WHEREAS, The National Council fully endorses the above resolution of the National Education Association, and

WHEREAS, The National Council of Teachers of Mathematics, an organization of teachers who are loyal and devoted to our system of government and society, deems it its duty to implement the above resolution of the National Education Association,

THEREFORE, Be it resolved that

First: The National Council of Teachers of Mathematics will deny any elective and appointive office (including committees) to those who are:

(a) Members of the Communist Party;
(b) Members of organizations which are known and which have been declared subversive;
(c) Members of organizations which have been declared Communist controlled and directed as well as members of organizations and associations which are affiliated with Communist controlled and Communist directed organizations.

Second: The Board of Directors of the National Council of Teachers of Mathematics is hereby empowered and directed to implement and enforce this resolution.

Dr. Christofferson moved that it be referred to the Board. General discussion followed: Mr. Charlesworth had previously conferred with Dr. Givens of N.E.A. concerning their policy. Mr. Urbancek then read the original N.E.A. policy, in full. This resolution follows:

Preservation of Democracy:

The National Education Association strongly asserts that all schools have an obligation to teach the rights, privileges, and the responsibilities of living in a democracy.

The responsibility of the schools is to teach the value of our American way of life, founded as it is on the dignity and worth of the individual; our youth should know it, believe in it, and live it continuously.

As a measure of defense against our most potent threat, our American schools should teach about communism and all forms of totalitarianism, including the principles and practices of the Soviet Union and the Communist Party in the United States. Teaching about communism does not mean advocacy of communism. Such advocacy should not be permitted in American schools.

Members of the Communist party shall not

(*Continued on page 446*)

Program

The Twelfth Christmas Meeting
The National Council of Teachers of Mathematics

Oklahoma Agricultural and Mechanical College
Stillwater, Oklahoma*
December 27, 28, 29, 1951

THURSDAY, DECEMBER 27, 1951

9:00 A.M.—12:00 NOON. Meeting of the Board of Directors—Meeting Room 9

2:00 P.M.–5:00 P.M. Meeting of the Board of Directors—Meeting Room 9

10:00 A.M.–5:00 P.M. Registration—Main Lobby, Student Union

7:30 P.M. Entertainment—Student Union

FRIDAY, DECEMBER 28, 1951

8:00 A.M.–5:00 P.M. Registration—Main Lobby, Student Union

The Friday morning meetings are centered around the theme, "The Place of Experimentation and Discovery on the Part of Pupils in the Learning Process in Mathematics."

9:00 A.M.–10:15 A.M. General Session—Ballroom

Presiding: HARRY W. CHARLESWORTH, President of the National Council of Teachers of Mathematics, East High School, Denver, Colorado

Helping Pupils Discover Mathematical Concepts, F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

10:30 A.M.–12:00 NOON. Elementary Section—Varsity Room

Presiding: EDITH STEANSON, University Elementary School, University of Oklahoma, Norman, Oklahoma

The How and Why of Discovery in Elementary School Arithmetic, ESTHER J. SWENSON, University of Alabama, University, Alabama

Some Practices Concerning Experimentation and Discovery in Learning in Mathematics in the Elementary School, W. I. LAYTON, Stephen F. Austin State College, Nacogdoches, Texas

10:30 A.M.–12:00 NOON. Junior High School Section—Pioneer Room

Presiding: AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

Student Discovery of Algebraic Concepts, OSCAR F. SCHAAF, University School, Ohio State University, Columbus, Ohio

Geometric Readiness. IRVIN H. BRUNE, Iowa State Teachers College, Cedar Falls, Iowa

10:30 A.M.–12:00 NOON. Senior High School Section—Howdy Room

Presiding: MARIE WILCOX, Washington High School, Indianapolis, Indiana

The Pupil Discovers Algebra, EUNICE LEWIS, University School, University of Oklahoma, Norman, Oklahoma

Experimentation and Discovery in Geometry, PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan

10:30 A.M.–12:00 NOON. College Section—Pow-wow Room

Presiding: GILBERT ULMER, University of Kansas, Lawrence, Kansas

Student Discovery on the College Level by Means of Construction of Models, C. B. READ, University of Wichita, Wichita, Kansas

Confidence, Confusion, Completion, RUTH LANE, Sam Houston State Teachers College, Huntsville, Texas

* All meetings will be held in Student Union Building.

10:30 A.M.–12:00 NOON. Teacher Education Section—Corral Room

Presiding: WILLIAM A. GAGER, University of Florida, Gainesville, Florida

Best Training for Future Teachers, ZEKE LOFLIN, Southwestern Louisiana Institute, Lafayette, Louisiana

Numerical Concepts of Prospective Elementary Teachers before Taking a Course in Professionalized Content in Arithmetic, ELLA MARTH, Harris Teachers College, St. Louis, Missouri

12:15 P.M.–1:30 P.M. Get-Acquainted Luncheon—Cafeteria

2:00 P.M.–4:00 P.M. *Elementary Section*—Varsity Room

Presiding: DALE PATTERSON, Colorado State College of Education, Greeley, Colorado

Panel Discussion: *The Role of Textbooks in Teaching Arithmetic in the Elementary School*, FOREST FISCH, Colorado State College of Education, Greeley, Colorado; JACK HALL, Laboratory School, State College of Education, Pittsburg, Kansas; LUCY L. ROSENQUIST, Colorado State College of Education, Greeley, Colorado (Emeritus)

2:00 P.M.–4:00 P.M. Secondary Section—Howdy Room

Classroom Experimentation Utilizing the Meaning-Discovery Approach to Mathematics. This section is sponsored by the Wichita Mathematics Association, Wichita, Kansas

Presiding: LUCY E. HALL, High School North, Wichita, Kansas

The Teacher's Role as Guide and Director, LOTTCHEN L. HUNTER, Wichita Public Schools, Wichita, Kansas

The Slide-Rule as a Means of Extending Number Concepts in Junior High School, DAVID L. HOLLAND, Robinson Intermediate School, Wichita, Kansas

New Approaches to Learning, T. OTHO COTT, Allison Intermediate School, Wichita, Kansas

The Discovery Approach to Geometry, CELIA CANINE, High School North, Wichita, Kansas

A Second Course in Basic Mathematics, KENNETH N. NICKEL, High School East, Wichita, Kansas

2:00 P.M.–4:00 P.M. College Section—Pow-wow Room

Presiding: C. B. READ, University of Wichita, Wichita, Kansas

What the College Mathematics Department Can Expect from Their Entering Freshmen, and What to Do about It, ESTHER F. GIBNEY, University of Houston, Houston, Texas

Our Neglect of Mechanics, C. E. SPRINGER, University of Oklahoma, Norman, Oklahoma

Learning Theory Applied to College Mathematics, LYLE J. DIXON, University of Kansas, Lawrence, Kansas

2:00 P.M.–4:00 P.M. Teacher Education Section—Corral Room

Presiding: H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio

What Do Good Mathematics Teachers Do Better than Poor Mathematics Teachers? ROBERT E. PINGRY, University of Illinois, Urbana, Illinois

Four Types of Number Activity that Teachers Should Develop. HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Mich.

A Program for Training Teachers of Arithmetic in the Primary Grades. EDWINA DEANS, Arlington Public Schools, Arlington, Virginia

2:00 P.M.–4:00 P.M. Research Section—Pioneer Room

This section is sponsored by the Research Committee of the National Council of Teachers of Mathematics.

Presiding: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

The Teaching of Statistics in Secondary School Mathematics, MAX BEBERMAN, University High School, University of Illinois, Urbana, Illinois

Mathematics in Meteorology, with Its Implications for the Teaching of Secondary and College Mathematics, MAX KRAMER, New Mexico College of

Agriculture and Mechanic Arts, State College, New Mexico

Influences in the Development of Geometry as a Subject in the Secondary School, G. H. LUNDBERG, Vanderbilt University, Nashville, Tennessee

4:00 P.M.–5:00 P.M. Affiliated Groups Section—Howdy Room

Presiding: JOHN MAYOR, University of Wisconsin, Madison, Wisconsin

Panel Discussion: *What the Affiliated Groups Are Doing,* REPRESENTATIVES OF THE GROUPS IN OKLAHOMA AND NEIGHBORING STATES

4:00 P.M.–5:00 P.M. Mathematics Films—Audio-Visual Center, Little Theater

6:30 P.M. Banquet—Ballroom

Twentieth Century Mathematics Teaching—Retrospect and Prospect, J. O. HASSLER, University of Oklahoma, Norman, Oklahoma

SATURDAY, DECEMBER 29, 1951

8:00 A.M.–3:30 P.M. Registration—Main Lobby

8:00 A.M.–9:00 A.M. Mathematics Films—Audio-Visual Center, Little Theater

9:00 A.M.–10:15 A.M. General Session—Ballroom

Presiding: JOHN MAYOR, University of Wisconsin, Madison, Wisconsin

The Evaluation of Learning in Mathematics, MAURICE HARTUNG, University of Chicago, Chicago, Illinois

The general session will be followed by discussion groups in which the topic of the address will be discussed with reference to the various instructional levels. Members are urged to register for the group at the level in which they are most interested.

10:30 A.M.–12:00 NOON. Discussion Groups

The Evaluation of Learning in Mathematics

Group 1. *In the Elementary School*—Meeting Room 2

Leader: GLENADINE GIBB, Iowa State Teachers College, Cedar Falls, Iowa

Group 2. *In the Junior High School*—Meeting Room 3

Leader: H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio

Group 3. *In the Senior High School*—Meeting Room 5

Leader: GILBERT ULMER, University of Kansas, Lawrence, Kansas

Group 4. *In the College*—Pow-wow Room

Leader: HAROLD TRIMBLE, Iowa State Teachers College, Cedar Falls, Iowa

Group 5. *In Teacher Education*—Meeting Room 6

Leader: NATHAN LAZAR, Ohio State University, Columbus, Ohio

10:30 A.M.–12:00 NOON. General Mathematics Section—Howdy Room

This section is sponsored by the Colorado Council of Teachers of Mathematics.

Presiding: FOREST FISCH, Colorado State College of Education, Greeley, Colorado

Trends and Improvements in the Teaching of General Mathematics, ALBERT N. RECHT, University of Denver, Denver, Colorado

RUTH IRENE HOFFMAN, Denver Public Schools, Denver, Colorado

C. F. BARR, University of Wyoming, Laramie, Wyoming

10:30 A.M.–12:00 NOON. General Section—Pioneer Room

Panel Discussion: *The National Emergency and the Teaching of Mathematics*

Chairman: JAMES H. ZANT, Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma

Participants: M. H. AHRENDT, Executive Secretary, National Council of Teachers of Mathematics; LYLE J. DIXON, University of Kansas, Lawrence, Kansas; L. W. LAVENGOOD, Tulsa Public Schools, Tulsa, Oklahoma

10:30 A.M.–12:00 NOON. Curriculum Section—Corral Room

This section is sponsored in part by the Colorado Council of Teachers of Mathematics.

Presiding: F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

The Unified Kindergarten through Twelfth Grade Mathematics Program of the Denver Public Schools, BURNETT SEVERSON, Denver Public Schools, Denver, Colorado

Recent State Curriculum Developments in Mathematics, DANIEL SNADER, University of Illinois, Urbana, Illinois

1:30 P.M.–3:30 P.M. Elementary Section—Varsity Room

This section is sponsored by the Dallas Elementary Mathematics Association, Dallas, Texas.

Presiding: CARRIE DENSON, Dallas Public Schools, Dallas, Texas

The Teaching of Long Division, RUTH WHELESS, Dallas Public Schools, Dallas, Texas

Discussion

1:30 P.M.–3:30 P.M. Secondary School Section—Howdy Room

This section is sponsored in part by the Beaumont Mathematics Association, Beaumont, Texas.

Presiding: KENNETH BROWN, University of Tennessee, Knoxville, Tennessee

Problem Solving and Mathematical Concepts in the Secondary School, ROSALIE DUNHAM, Wilson Junior High School, Tulsa, Oklahoma

An Aid in the Development of Independent, Orderly, and Logical Proofs, VELMA B. OAKES, Siloam Springs High School, Siloam Springs, Arkansas

Panel Discussion: *Aids in Teaching High School Mathematics*, ALICE MCCALL, Beaumont High School, Beaumont, Texas; NATALIE DINAN, Beaumont High School, Beaumont, Texas; KATHRENE BAILEY, South Park High School, Beaumont, Texas; PEARL BOND, Beaumont High School, Beaumont, Texas

1:30 P.M.–3:30 P.M. College Section—Pow-wow Room

This section is sponsored by the Arkansas Council of Mathematics Teachers.

Presiding: OLIN HUGHES, Arkansas State Teachers College, Conway, Arkansas

The Faculty Looks at Mathematics for General Education, H. H. HYMAN, Florida State University, Tallahassee, Florida

Mathematics Intangibles, JOHN ABERNATHY, Arkansas Polytechnic College, Russellville, Arkansas

1:30 P.M.–3:30 P.M. Teacher Education Section—Corral Room

Theme: *Advancing the Cause of Mathematical Education*

Presiding: NATHAN LAZAR, Ohio State University, Columbus, Ohio

Improving the Training of Teachers of Secondary Mathematics, A. C. MADDOX, Northwestern State College, Natchitoches, Louisiana

Improving the Training of Teachers of Elementary Mathematics, F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–3:30 P.M. Enrichment Section—Pioneer Room

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

Books for Mathematics—Recreational and Supplementary Reading, MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois

The Mathematics of Crystals, AARON BAKST, Flushing, New York

Mathematics for the Most Capable Students, H. W. CHARLESWORTH, East High School, Denver, Colorado

PROGRAM COMMITTEE

Chairman, Lenore John; Ruth Clough, Mary Lee Foster, Ida Mae Heard, Agnes Herbert, E. H. C. Hildebrandt, Jessie May Hoag, L. W. Lavengood, John R. Mayor, Philip Peak, Henry Van Engen, Marie Wilcox, James H. Zant.

ANNOUNCEMENTS

Registration

The registration fee is fifty cents for

members of The National Council of Teachers of Mathematics, members of the Mathematical Association of America, and teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commercial exhibitors are not charged a registration fee but should register. You are urged to register in advance, using the Advance Registration and Reservation Form below. Registration will be in the Main Lobby of the Student Union Building.

Room Reservations

The Union Club, an integral part of the Student Union, will accommodate a limited number of guests in regular hotel rooms, each with bath, at the following rates.

One person to a room	$4.25
Two persons to a room	6.25
Three persons to a room	7.75
Four persons to a room	9.25
A dormitory holding 20 people	2.00 per person

Reservations for rooms in the Union Club should be sent directly to Mr. Arthur H. Taylor, The Union Club, Oklahoma A. & M. College, Stillwater, Okla.

The following hotels and motor courts in Stillwater are available: Going Hotel, Grand Hotel, Aggie Courts, Anglin Motel, Curran-Au-Tel, Rosa Courts and Sloan's Tourist Courts. Reservations for rooms in these should be sent directly to the hotel or motor court.

Any overflow will be housed in the College Dormitories.

Food Service

The Cafeteria in the basement of the Student Union Building will serve meals from Thursday evening, December 27, through Saturday noon, December 29. The Student Union Coffee Shop will also be open for all meals.

The Banquet will be held Friday evening, December 28, in the Ballroom. Reservations ($2.50) should be made in advance on the Registration and Reservation Form.

The Get-Acquainted Luncheon Friday noon, December 28, will be cafeteria style. No reservation is necessary.

Discussion Groups

Please register in advance for the Saturday morning discussion groups, so that plans may be made for additional groups if the registration is large. Please use the Registration and Reservation Form and indicate first and second choices.

Entertainment and Recreation

Since the Council will have the complete facilities of the Student Union Building at its disposal, all entertainment and recreation will be arranged within the building.

Exhibits

There will be exhibits of mathematical models, instruments, teaching aids and other classroom materials. Teachers are invited to bring materials for exhibit. Textbooks and commercial teaching aids will also be on exhibit. Inquiries should be addressed to James H. Zant, Department of Mathematics, Oklahoma A. &. M. College, Stillwater, Oklahoma.

Supplies and Equipment

Speakers and other participants in the program who need projection equipment or other materials should communicate not later than December 10 with James H. Zant, Oklahoma A. & M. College, Stillwater, Oklahoma.

Films and Film Strips

Mathematics films will be shown as indicated in the program. Persons wishing to suggest films to be shown should communicate, before December 1, with Lenore John, Laboratory School, University of Chicago, Chicago, Illinois.

Transportation

Stillwater cannot be reached efficiently by rail. Those who come from Chicago

ADVANCE REGISTRATION AND RESERVATION FORM

Fill out completely and mail to Mr. Raymond L. Caskey, Department of Mathematics, Oklahoma A. & M. College, Stillwater, Oklahoma, before December 10, 1951.

Mr., Miss, Mrs. ____
Last name First name Initial

Address ____
Street and number City State

Registering as member of N.C.T.M. __________; M.A.A. __________; Student __________.

Elementary teacher __________; Exhibitor __________; Non-member __________. Members of families and friends of those attending the meetings are welcome to all social and recreational events.

Registration fee: $.50 or $1.50 Amount __________

Room Registration

For the Union Club, send directly to
Mr. Arthur H. Taylor, Union Club, Oklahoma A. & M. College,
Stillwater, Oklahoma

__________ Banquet Reservations at $2.50 each __________

Total $__________

Make checks payable to Raymond L. Caskey

Discussion Group Registration
State discussion group number

First Choice: Group __________

Second Choice: Group __________

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and the East and North should purchase tickets to Perry, Oklahoma, which is 25 miles from Stillwater. When bus service is not available, transportation from Perry to Stillwater will be furnished by the College, provided notice is received by the Department of Mathematics before December 15, if possible. Those arriving at Oklahoma City or Tulsa will find bus service available. It is probable that air service will be available to Stillwater by December 1; at present air passengers must travel from Oklahoma City or Tulsa by bus. Persons driving their own cars will find excellent highways leading into Stillwater from all directions.

Minutes

(*Continued from page 440*)

be employed in our schools. Communist organizations and communist front organizations should be required by law to register with the Attorney-General of the United States.

The Association charges the teaching profession with the obligation of providing the best defense of democracy through full participation in making democracy really live and work.

The Association again reminds the public of the repeated pronouncements of our military leaders and statesmen that education is the basis of our national security and that a well-planned, adequately supported system of free public schools is fundamental to the perpetuation of our American way of life.

Mr. Belding moved that this Body go on record concurring with the N.E.A. Resolution. This was seconded by Mr. Urbancek and carried.

There being no further business the meeting adjourned.

Respectfully submitted,
AGNES HERBERT, *Acting Secretary*